Supersymmetric models for the LHC

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Abstract

The aim of these lectures is to give an elementary introduction to the techniques and tools of supersymmetric model building. The reader is assumed to be somewhat familiar with the basics of supersymmetry and with the MSSM.
Warning: These notes are preliminary, and likely full of sign errors, lost factors of 2 and of i, typos and so on. I hope they will eventually evolve into being genuinely useful, but until then, use them at your own risk!

1 Introduction

Weak-scale supersymmetry (SUSY) is still one of the most promising scenarios for new physics at the TeV scale. It can solve the electroweak Hierarchy Problem, provide a realistic dark matter candidate, and allow for the unification of the fundamental forces of the Standard Model.

Supersymmetry must be spontaneously broken if it is to be realised in Nature. Sum rules relating the masses of the Standard Model (SM) particles and their supersymmetric partners decree that supersymmetry breaking should take place in a *hidden sector*, which cannot have any tree-level renormalizable couplings to the Standard Model, since otherwise some of the superpartners would have long been observed. To study the phenomenology of the Supersymmetric Standard Model (SSM), the effects of hidden-sector supersymmetry breaking are parameterised by explicit SUSY-breaking *soft terms*, which appear in the effective SSM when integrating out the hidden sector dynamics.

In the simplest and most frequently studied incarnation of the SSM, the *Minimal Supersymmetric Standard Model with R-parity* (MSSM), the soft SUSY-breaking terms contain around 100 independent free parameters. Even more free parameters appear when the Higgs sector is extended, or when the requirement of exact R-parity conservation is relaxed. Given a model for the hidden sector, and for SUSY breaking mediation to the visible sector, these parameters will be fixed. However, a multitude of such models have been proposed, and it is difficult to assess which among these are the more promising or potentially realistic ones.

In these lectures we will review how soft terms can be calculated from first principles, given a model for SUSY breaking mediation and/or SUSY breaking. The aim is to bridge the gap between hard-core theorists, working on formal supersymmetry or string theory, and phenomenologists interested primarily in the LHC phenomenology.

2 Common benchmark scenarios, and why we shouldn't believe in them

Recall the field content of the Minimal Supersymmetric Standard Model, in terms of representations of SU(3) × SU(2) × U(1):

spin- 0	1/2	1	SU(3)	SU(2)	U(1)
	λ_1	B_μ	**1**	**1**	0
	λ_2	W_μ	**1**	**3**	0
	λ_3	G_μ	**8**	**1**	0
$\widetilde{Q}_I$	Q_I		**3**	**2**	1/6
$\widetilde{U}_I$	U_I		$\overline{\mathbf{3}}$	**1**	−2/3
$\widetilde{D}_I$	D_I		$\overline{\mathbf{3}}$	**1**	1/3
$\widetilde{L}_I$	L_I		**1**	**2**	−1/2
$\widetilde{E}_I$	E_I		**1**	**1**	1
H_u	$\widetilde{H}_u$		**1**	**2**	1/2
H_d	$\widetilde{H}_d$		**1**	**2**	−1/2

Here the generation index I runs from 1 to 3. Each row represents a supersymmetric multiplet, with its components related by supersymmetry transformations.

The MSSM Lagrangian can be written as

$$\mathcal{L}_{\rm MSSM} = \mathcal{L}_{\rm kinetic} + \mathcal{L}_{\rm Yukawa} - V_{\rm SUSY} + \left(\mu\, \widetilde{H}_u \widetilde{H}_d + \text{h.c.}\right) + \mathcal{L}_{\rm soft}\,, \tag{1}$$

where $\mathcal{L}_{\rm kinetic}$ contain the gauge-kinetic terms for quarks, leptons, gauge and Higgs bosons and their superpartners, $\mathcal{L}_{\rm Yukawa}$ contains Yukawa interactions, $V_{\rm SUSY}$ contains quartic interactions between the squarks, sleptons and Higgs fields as well as a supersymmetric Higgs mass term, the $\mu\, \widetilde{H}_u \widetilde{H}_d$ term is the corresponding supersymmetric Higgsino mass, and

$$\begin{aligned}
\mathcal{L}_{\rm soft} = &-\frac{1}{2}\sum_{a=1}^{3} M_a\, \lambda^a \lambda^a + \text{h.c.} \\
&+ \sum_{I,J=1}^{3} \left(m^2_{QIJ}\, \widetilde{Q}_I^\dagger \widetilde{Q}_J + m^2_{UIJ}\, \widetilde{U}_I^\dagger \widetilde{U}_J + m^2_{DIJ}\, \widetilde{D}_I^\dagger \widetilde{D}_J + m^2_{LIJ}\, \widetilde{L}_I^\dagger \widetilde{L}_J + m^2_{EIJ}\, \widetilde{E}_I^\dagger \widetilde{E}_J \right) \\
&+ \sum_{I,J=1}^{3} \left(a_{UIJ}\, H_u \widetilde{Q}_I \widetilde{U}_J + a_{DIJ}\, H_d \widetilde{Q}_I \widetilde{D}_J + a_{EIJ}\, H_d \widetilde{L}_I \widetilde{E}_J + \text{h.c.} \right) \\
&+ m^2_{H_u} |H_u|^2 + m^2_{H_d} |H_d|^2 + (b\, H_u H_d + \text{h.c.})
\end{aligned} \tag{2}$$

contains the SUSY-breaking soft terms. In detail, $M_{1,2,3}$ are soft masses for the U(1), SU(2) and SU(3) gauginos respectively; $m^2_{\Phi IJ}$ with $\Phi = Q, U, D, L, E$ are (hermitian) scalar soft mass matrices for the three generations of squarks and sleptons; $a_{\Phi IJ}$ are scalar trilinear couplings; and $m^2_{H_{u,d}}$ and b are SUSY-breaking Higgs mass parameters.

The soft parameters should be understood as running parameters: When including quantum effects, they depend on the renormalisation scale. In particular, the soft terms at the TeV scale (which determine the physical particle masses) are in general different from the soft terms at some high UV completion scale where they are generated.

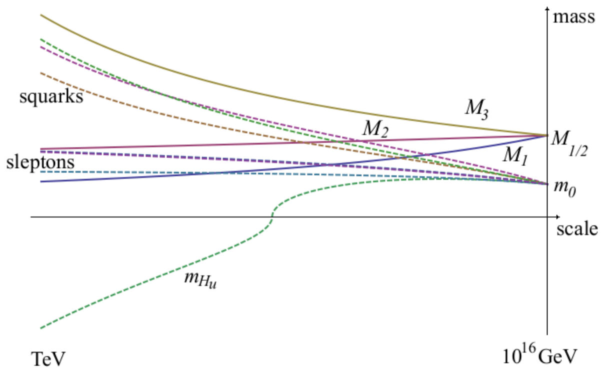

Figure 1: Sketch of the RG evolution of some MSSM mass parameters between the GUT scale and the TeV scale. Dashed lines are scalar masses, solid lines are gaugino masses.

2.1 The CMSSM

The *Constrained MSSM* is defined by the following soft parameters at the scale of grand unification $M_{\mathrm{GUT}} \approx 2 \cdot 10^{16}$ GeV:

- $m^2_{H_u} = m^2_{H_d} \equiv m^2_0$,
- $m^2_{QIJ} = m^2_{UIJ} = m^2_{DIJ} = m^2_{LIJ} = m^2_{EIJ} = m^2_0\,\delta_{IJ}$,
- $M_1 = M_2 = M_3 \equiv M_{1/2}$,
- $a_{UIJ} = A_0\,y_{UIJ}$, $a_{DIJ} = A_0\,y_{DIJ}$, $a_{EIJ} = A_0\,y_{EIJ}$, where $y_{U,D,E}$ are the Yukawa matrices.

In other words, there are only three dimensionful parameters: m_0, governing the scalar soft masses; $M_{1/2}$, governing the gaugino masses; and A_0, governing the trilinear couplings. There are two more dimensionful parameters in the MSSM which are not fixed by this prescription, namely the supersymmetric Higgsino mass μ and the soft Higgs mass mixing b. In the CMSSM these two are adjusted such that the electroweak symmetry is broken at the proper scale, with a Z boson mass $m_Z = 91.2$ GeV. This leaves an additional free parameter β, which is related to the ratio of up-type and down-type Higgs expectation values as

$$\tan\beta = \langle H_u\rangle / \langle H_d\rangle\,. \qquad (3)$$

In addition, while μ can be chosen real without loss of generality, its sign is undetermined by these conditions and must be chosen to be either $+1$ or -1.

The soft terms must be evolved to the TeV scale according to their renormalisation group equations. A typical RG evolution is sketched in Fig. 1. Characteristically, the up-type Higgs mass becomes negative at low energies thanks to large loop contributions from the top Yukawa coupling, signalling the breaking of electroweak symmetry.

Usual motivation: The CMSSM soft term pattern supposedly follows from “minimal supergravity”, where SUSY breaking is dominated by a single hidden-sector chiral multiplet and mediated by Planck-suppressed interactions within supergravity.

Some caveats: The "minimal supergravity" picture itself is poorly motivated, and does not actually lead to these soft term patterns. Several extra assumptions are needed to obtain the CMSSM, and some of them lack a good theoretical justification. The CMSSM should be regarded as a useful benchmark scenario to get a rough idea of the MSSM phenomenology, but not as a realistic model. We will return to this on a more technical level in Section 6 in these lectures.

2.2 GMSB

(Minimal) gauge-mediated SUSY breaking is defined by

- $M_a = \frac{g_a^2}{16\pi^2}\, n\, \Lambda$,
- $m_\Phi^2 = 2n\Lambda^2 \sum_{a=1}^3 C_a(\Phi) \left(\frac{g_a^2}{16\pi^2}\right)^2$,
- $a_{U,D,E} = 0$.

Here $g_{1,2,3}$ are the gauge couplings, $n \in \mathbb{N}$ is a discrete free parameter, Λ is a continuous free parameter of dimension one, and $C_a(\Phi)$ is a group-theoretic factor which depends on the gauge quantum numbers of $\Phi = Q, U, D, L, E, H_u, H_d$. We have omitted generation indices because the squark and slepton soft masses are again diagonal and universal in flavour space. All these soft terms are defined at a scale M, which is another free parameter of the model subject to $\Lambda < M < M_{\mathrm{GUT}}$. Again μ and b are adjusted to reproduce the correct electroweak symmetry breaking scale, leaving $\tan\beta$ and $\mathrm{sign}(\mu)$ as additional free parameters. In summary, the GMSB spectrum is fixed by two mass parameters Λ and M, together with $\tan\beta$ and the discrete parameters n and $\mathrm{sign}(\mu)$.

Usual motivation: These soft terms are asserted to follow from "minimal gauge mediation" independently of the details of the hidden sector. In MGM, n copies of *messenger fields* in the $\mathbf{5} \oplus \overline{\mathbf{5}}$ representation of $\mathrm{SU}(5) \supset \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ couple to a hidden-sector singlet via Yukawa-like interactions. They are given supersymmetric masses M and SUSY-breaking mass splittings Λ. Messenger loops induce the above soft terms at leading order in Λ/M.

Some caveats: Again this depends on extra assumptions on the hidden sector. While it may come closer to a realistic model than the CMSSM, in general gauge-mediated models are much more complicated and allow for significantly greater freedom in the soft terms. This will be the topic of Section 5 of these lectures.

2.3 AMSB

For completeness, *(minimal) anomaly-mediated SUSY breaking* is defined by

- $M_a = m_{3/2} \frac{\beta_{g_a}}{2g_a}$,
- $m_\Phi^2 = -\frac{1}{4} m_{3/2}^2 \frac{\mathrm{d}}{\mathrm{d}t}\gamma_\Phi$,
- $a_{U,D,E} = -\frac{1}{2} m_{3/2} \left(\gamma_{Q,L} + \gamma_{U,D,E} + \gamma_{H_u,H_d}\right) y_{U,D,E}$.

where the βs and γs are the beta functions and anomalous dimensions, $t \equiv \log Q$, Q is the renormalisation scale, and the γs entering the trilinear couplings correspond to the relevant fields in Eq. (2). The parameter $m_{3/2}$ is the gravitino mass. As previously, μ and b are fixed up to sign(μ) by $\tan\beta$ and m_Z.

Usual motivation: This soft term pattern is predicted if the hidden sector couples truly minimally, i.e. only through four-dimensional supergravity, to the SSM. (In that sense, it should really be called "gravity mediation", but that term is usually reserved for any generic mediation mechanism which employs gravitationally suppressed interactions.)

Some caveats: As in gauge mediation, μ and b are not actually predicted by the model, and generating them requires additional assumptions. Furthermore, one of the predictions of AMSB is unfortunately that $m_E^2 < 0$, since $\dot{\gamma}_E < 0$. The usual phenomenological fix for this is to add a universal scalar soft mass $m_0^2 > 0$ to all soft scalar masses, but this has no theoretical justification. Thus anomaly mediation may play a role for the SUSY breaking soft terms, but it is not likely to be the dominant one, and we will have nothing more to say about it in these lectures.