3 Superfield formalism

Superfields can greatly simplify calculations in supersymmetric field theories while keeping supersymmetry manifest. A superfield is a function of *superspace*, which is Minkowski space-time augmented with additional fermionic coordinates θ^α and $\bar{\theta}_{\dot{\alpha}}$:

$$\Phi = \Phi\left(x^\mu, \theta^\alpha, \bar{\theta}_{\dot{\alpha}}\right) . \tag{4}$$

Here θ is a left-handed two-component spinor with anticommuting components $\left(\theta^{(1)}, \theta^{(2)}\right)$:

$$\begin{aligned} \theta^{(1)}\theta^{(2)} &= -\theta^{(2)}\theta^{(1)} , \\ \theta^{(1)}\theta^{(1)} &= \theta^{(2)}\theta^{(2)} = 0 . \end{aligned} \tag{5}$$

Define

$$\theta^2 \equiv \theta^\alpha\theta^\beta\epsilon_{\alpha\beta} = \theta^{(1)}\theta^{(2)} - \theta^{(2)}\theta^{(1)} = 2\theta^{(1)}\theta^{(2)} \tag{6}$$

(here the superscript in θ^2 is an exponent, not an index!). The right-handed $\bar{\theta}$ coordinate satisfies similar relations, and its components also anticommute with those of θ:

$$\theta^\alpha\bar{\theta}_{\dot{\beta}} = -\bar{\theta}_{\dot{\beta}}\theta^\alpha . \tag{7}$$

Note that all expressions containing more than two powers of components of either θ or $\bar{\theta}$ vanish. This allows to write down a finite expansion of any superfield Φ in powers of θ and $\bar{\theta}$:

$$\begin{aligned} \Phi\left(x^\mu, \theta^\alpha, \bar{\theta}_{\dot{\alpha}}\right) =& f(x^\mu) + \theta^\alpha\psi_\alpha(x^\mu) + \bar{\theta}_{\dot{\alpha}}\bar{\xi}^{\dot{\alpha}}(x^\mu) + \theta^2\, g(x^\mu) + \bar{\theta}^2\, h(x^\mu) + \theta^\alpha\sigma^{\nu}_{\alpha}{}^{\dot{\alpha}}\bar{\theta}_{\dot{\alpha}}v_\nu(x^\mu) \\ &+ \theta^2\bar{\theta}_{\dot{\alpha}}\bar{\lambda}^{\dot{\alpha}} + \bar{\theta}^2\theta^\alpha\chi_\alpha(x^\mu) + \theta^2\bar{\theta}^2 m(x^\mu) . \end{aligned} \tag{8}$$

Here f, g, h, and m are ordinary scalar fields; ψ, $\bar{\xi}$, χ, and $\bar{\lambda}$ are two-component spinor fields; v_ν is a vector field; and σ^ν has components $(\mathbb{1}, \sigma^i)$ (with σ^i the Pauli matrices) such as to transform as a vector under Lorentz transformations.

Under supersymmetry transformations the component fields of Φ will transform into each other. However, the general superfield of Eq. (8) corresponds to a reducible representation of supersymmetry: Not all the components are needed to form a supermultiplet. To construct reducible representations, one imposes supercovariant *constraints*.

3.1 Chiral superfields

The most important example of an irreducible representation is the *chiral superfield*, which contains a complex scalar and a two-component spinor as propagating degrees of freedom. A chiral superfield is defined to satisfy the constraint

$$\boxed{\overline{D}^{\dot{\alpha}}\Phi = 0} \tag{9}$$

where

$$\overline{D}^{\dot{\alpha}} = -\frac{\partial}{\partial\theta_{\dot{\alpha}}} - i\theta^\alpha\sigma^{\mu}_{\alpha}{}^{\dot{\alpha}}\frac{\partial}{\partial x^\mu} . \tag{10}$$

It has a component expansion

$$\boxed{\begin{aligned}\Phi\left(x^\mu,\theta^\alpha,\bar\theta_{\dot\alpha}\right) &= \varphi(x^\mu)+\sqrt{2}\theta^\alpha\psi_\alpha(x^\mu)+\theta^2\,F(x^\mu)\\ &+\text{(space-time derivatives acting on }\varphi\text{ and }\psi)\end{aligned}} \tag{11}$$

in terms of a complex scalar φ, a two-component left-handed spinor ψ, and a complex scalar auxiliary field F which is not a propagating field. The conjugate $\overline{\Phi}$ of Φ will then satisfy the defining constraint for an *anti-chiral superfield*,

$$D_\alpha\overline{\Phi}=0\,; \tag{12}$$

with

$$D_\alpha=\frac{\partial}{\partial\theta^\alpha}+i\sigma^{\mu\,\dot\alpha}_{\alpha}\bar\theta_{\dot\alpha}\frac{\partial}{\partial x^\mu}\,. \tag{13}$$

The most general supersymmetric two-derivative Lagrangian involving only chiral supermultiplets Φ_i can then be written as

$$\boxed{\mathcal{L}=K\left(\Phi_i,\overline{\Phi}_j\right)\big|_{\theta^2\bar\theta^2}+W\left(\Phi_i\right)\big|_{\theta^2}+\overline{W}\left(\overline{\Phi}_j\right)\big|_{\bar\theta^2}\ .} \tag{14}$$

Here $K|_{\theta^2\bar\theta^2}$ denotes the coefficient of $\theta^2\bar\theta^2$ in an expansion of K in θ and $\bar\theta$, and correspondingly for $W|_{\theta^2}$ and $\overline{W}\big|_{\bar\theta^2}$. In Eq. (14), K is a real function of the chiral superfields and their conjugates, the *Kähler potential*. W is a holomorphic function of the chiral superfields only, the *superpotential*. $\mathcal{L}$ describes a renormalizable field theory if and only if K is quadratic and W is at most cubic.

One can also write the above projections on the θ^2 and $\theta^2\bar\theta^2$ components in terms of (formally defined) integrals over Grassmann coordinates. In the following, we will thus use the notation

$$\int \mathrm{d}^2\theta\,\mathrm{d}^2\bar\theta\,K\equiv K|_{\theta^2\bar\theta^2} \tag{15}$$

and

$$\int \mathrm{d}^2\theta\,W\equiv W|_{\theta^2}\ . \tag{16}$$

The canonical example for a supersymmetric model is the *Wess-Zumino model* of a single chiral superfield Φ. It is defined by

$$K=\overline{\Phi}\Phi,\qquad W=\frac{m}{2}\Phi^2+\frac{\lambda}{3}\Phi^3\,. \tag{17}$$

Taking the parameters λ and m to be real, and plugging the expansion Eq. (11) into the expression for the Lagrangian,

$$\mathcal{L}=\int\mathrm{d}^2\theta\,\mathrm{d}^2\bar\theta\ \overline{\Phi}\Phi+\int\mathrm{d}^2\theta\left(\frac{m}{2}\Phi^2+\frac{\lambda}{3}\Phi^3\right)+\int\mathrm{d}^2\bar\theta\left(\frac{m}{2}\overline{\Phi}^2+\frac{\lambda}{3}\overline{\Phi}^3\right)\,, \tag{18}$$

one obtains, after some spinor index reshuffling,

$$\begin{aligned}\mathcal{L}=\overline{F}F+m\varphi F+\lambda\varphi^2F-\frac{m}{2}\psi\psi-\lambda\phi\psi\psi+m\overline{\varphi}\overline{F}+\lambda\overline{\varphi}^2\overline{F}-\frac{m}{2}\overline{\psi\psi}-\lambda\overline{\varphi}\overline{\psi\psi}\\ +\text{(space-time derivatives acting on }\varphi\text{ and }\psi)\ .\end{aligned} \tag{19}$$

The field F enters algebraically and can therefore be eliminated by its equation of motion, which reads

$$0 = \frac{\partial \mathcal{L}}{\partial F} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu F)} = \frac{\partial \mathcal{L}}{\partial F} = \overline{F} + m\varphi + \lambda\varphi^2 \,. \tag{20}$$

Thus,

$$\overline{F} = -m\varphi - \lambda\varphi^2, \qquad F = -m\overline{\varphi} - \lambda\overline{\varphi}^2 \,. \tag{21}$$

Substituting this back into Eq. (19), and figuring out the derivative terms in detail, leaves one with

$$\mathcal{L} = -\partial_\mu\varphi\partial^\mu\overline{\varphi} + i\overline{\psi}\sigma^\mu\partial_\mu\psi - \frac{m}{2}(\psi^2 + \overline{\psi}^2) - \lambda\varphi\psi^2 - \lambda\overline{\varphi}\overline{\psi}^2 - |m\varphi - \lambda\varphi^2|^2. \tag{22}$$

This is the Lagrangian for ϕ^4 theory of a massive complex scalar φ, which couples by Yukawa interactions to a massive two-component spinor ψ. As a consequence of SUSY, there are relations between the couplings; e.g. the scalar and spinor masses are equal.

3.2 Real superfields

Real superfields satisfy the constraint $\overline{V} = V$ (or more generally $V^\dagger = V$ for matrix-valued real superfields). They contain the degrees of freedom for a gauge field and its gaugino superpartner. Super-gauge transformations can be used to bring its component expansion into the form

$$\boxed{V(x^\mu, \theta^\alpha, \bar{\theta}_{\dot{\alpha}}) = -\theta^\alpha \sigma^{\nu\,\dot{\alpha}}_{\alpha} \bar{\theta}_{\dot{\alpha}} A_\nu(x^\mu) + i\theta^2\bar{\theta}\bar{\lambda} - i\bar{\theta}^2\theta\lambda + \frac{1}{2}\theta^2\bar{\theta}^2\,\mathcal{D}} \tag{23}$$

(this is called *Wess-Zumino gauge*). While A_ν and λ are propagating fields, $\mathcal{D}$ turns out to be another auxiliary field. The D and $\overline{D}$ operators of Eqns. (13) and (10) may be used to define a chiral superfield W^α:

$$W^\alpha \equiv -\frac{1}{4}\overline{D}^{\dot{\alpha}}\overline{D}_{\dot{\alpha}} D^\alpha V \,. \tag{24}$$

It is then straightforward to write down a kinetic Lagrangian for A_ν and λ:

$$\mathcal{L} = \frac{1}{4}\int \mathrm{d}^2\theta\, W^\alpha W_\alpha + \frac{1}{4}\int \mathrm{d}^2\bar{\theta}\, \overline{W}_{\dot{\alpha}}\overline{W}^{\dot{\alpha}} = \frac{1}{2}\mathcal{D}^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - i\lambda\sigma^\mu\partial_\mu\bar{\lambda} \,. \tag{25}$$

The auxiliary field $\mathcal{D}$ is zero by its equation of motion, and the rest of $\mathcal{L}$ is the Lagrangian for a U(1) gauge field and a free Majorana spinor, the gaugino.

With chiral and real superfields, we can easily write down the Lagrangian for SUSY QED: Assign a charge q to each chiral superfield Φ, and replace the $\overline{\Phi}\Phi$ terms in the Kähler potential by $\overline{\Phi}e^{qV}\Phi$. Also make sure that the superpotential W is gauge invariant under $\Phi \to e^{iq\Lambda}\Phi$.

More generally, the most general two-derivative Lagrangian for a supersymmetric abelian gauge theory coupled to chiral superfields can be written

$$\boxed{\begin{aligned} \mathcal{L} = &\int \mathrm{d}^2\theta\,\mathrm{d}^2\bar{\theta}\; K\left(\overline{\Phi}_i, e^{q^a_{(j)}V^a}\Phi_j\right) + \int \mathrm{d}^2\theta\, W(\Phi_i) + \text{h.c.} \\ &+ \frac{1}{4}\int \mathrm{d}^2\theta\; S_{ab}(\Phi_i) W^{a\alpha} W^b_\alpha + \text{h.c.} \quad \left[+\xi_a \int \mathrm{d}^2\theta\,\mathrm{d}^2\bar{\theta}\; V_a\right] . \end{aligned}} \tag{26}$$

Here a, b label the gauge factors, and $q^a_{(i)}$ are the corresponding charges of the chiral superfields Φ_i. The Kähler potential K should be real and the superpotential W should be holomorphic as before. The *gauge-kinetic function* S_{ab} should be a holomorphic function of the chiral superfields. Finally, the last term in square brackets, the *Fayet-Iliopoulos term*, is a peculiarity of U(1) theories which will not be of interest for us. Now $\mathcal{L}$ is renormalizable if and only if K is quadratic, W is at most cubic, and S is constant.

All this is easily generalised to the case of non-abelian gauge groups, upon promoting the gauge superfields to take their values in some Lie algebra, $V = V^A T^A$, defining

$$W^\alpha = \overline{DD} e^{-V} D^\alpha e^V \,, \tag{27}$$

and assigning appropriate representations to the chiral superfields.

3.3 Supergravity

For later reference we also quote the embedding of Eq. (26) into supergravity. There are various different formulations for this, so we will pick the most convenient one, employing flat θ coordinates. It is convenient to define a real function

$$\Omega = -3\, M_P^2 \, \exp\left(-\frac{K}{3M_P^2}\right) \,. \tag{28}$$

We also need an auxiliary chiral superfield ϕ, called the *chiral compensator*, which by a gauge choice may be set to

$$\phi = \phi_0 \left(1 + F_\phi \theta^2\right) \,; \tag{29}$$

here ϕ_0 is an arbitrary dimensionless nonzero constant, often chosen to be 1, and F_ϕ is a dimension-one scalar auxiliary field associated to the supergravity multiplet. The Lagrangian is (dropping the FI term, and with derivatives understood to be made covariant)

$$\begin{aligned}\mathcal{L} = & \int \mathrm{d}^2\theta \, \mathrm{d}^2\bar\theta \, |\phi|^2 \, \Omega\left(\overline{\Phi}_i, e^{q^a_{(j)} V^a} \Phi_j\right) + \int \mathrm{d}^2\theta \, \phi^3 \, W(\Phi_i) + \text{h.c.} \\ & + \frac{1}{4} \int \mathrm{d}^2\theta \, S_{ab}(\Phi_i) W^{a\alpha} W^b_\alpha + \text{h.c.} - \frac{1}{6} |\phi|^2 \Omega \Big|_{\theta=\bar\theta=0} R \\ & + \text{vector auxiliary and gravitino terms}\,.\end{aligned} \tag{30}$$

The flat limit Eq. (26) is recovered by Weyl-rescaling the metric into Einstein frame and sending $M_P \to \infty$.

4 Soft parameters from superfield Lagrangians

We assume *F-term breaking* in a hidden sector, i.e. supersymmetry is broken by some hidden sector chiral superfields X_a acquiring non-vanishing vacuum expectation values for their F-term components F_a in Eq. (11). Hence the effects of SUSY breaking are encoded in

$$\langle X_a \rangle = F_a \theta^2 \tag{31}$$

with $F_a \neq 0$.

We will further assume that there are some mediator states which couple the visible and hidden sectors, and which acquire large supersymmetric masses of order $M \gg F_a$. This allows us to integrate out the messengers supersymmetrically, and to deal with the effects of SUSY breaking within a supersymmetric effective field theory at scales below M. For gravity mediation, $M = M_P$ is the Planck scale, whereas for gauge mediation, M is in general lower.

4.1 Scalar soft masses

We wish to derive the scalar soft masses in the rigid limit from Eq. (26). Denote by Φ_i the MSSM chiral superfields, and neglect the gauge sector for the moment. Integrating out the messenger states will result in an effective Kähler potential for the Φ_i:

$$K = K_{\text{hid.}}(X_a, \overline{X}_a) + \widetilde{K}\left(\frac{X_a}{M}, \frac{\overline{X}_a}{M}, \Phi_i, \overline{\Phi}_i\right) . \tag{32}$$

We can expand $\widetilde{K}$ to second order in the visible sector fields:

$$K = K_{\text{hid.}}(X_a, \overline{X}_a) + \sum_{ij} \mathcal{Z}_{ij}\left(\frac{X_a}{M}, \frac{\overline{X}_a}{M}\right) \overline{\Phi}_i \Phi_j + \mathcal{H}_{ij}\left(\frac{X_a}{M}, \frac{\overline{X}_a}{M}\right) \Phi_i \Phi_j + \text{h.c.} + \ldots \tag{33}$$

Now pick a basis of visible sector fields where $\mathcal{Z}$ is diagonal, $\mathcal{Z}_{ij} = \mathcal{Z}_i \delta_{ij}$. The term

$$\mathcal{L}_{\text{mass}} = \int \mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta}\, \sum_i \mathcal{Z}_i\left(\frac{X_a}{M}, \frac{\overline{X}_a}{M}\right) |\Phi_i|^2 \tag{34}$$

induces soft masses for the scalar components of the Φ_i superfields. To see this, let us expand the integrand in θ and $\bar{\theta}$:

$$\begin{aligned}\mathcal{L}_{\text{mass}} = \int \mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta}\, \sum_i \left(\mathcal{Z}_i\Big|_0 + \frac{\partial \mathcal{Z}_i}{\partial X_a}\Big|_0 F_a\theta^2 + \text{h.c.} + \frac{\partial^2 \mathcal{Z}_i}{\partial X_a \partial \overline{X}_b}\Big|_0 F_a \overline{F}_b \theta^2\bar{\theta}^2\right) \times \\ \times \left(|\varphi_i|^2 + F_i \overline{\varphi}_i \theta^2 + \text{h.c.} + |F_i|^2\, \theta^2\bar{\theta}^2\right) + \text{fermions} + \text{space-time derivatives}\,.\end{aligned} \tag{35}$$

Here $\mathcal{Z}_i\Big|_0$ denotes the $\theta = \bar{\theta} = 0$ component of $\mathcal{Z}_i$, and we have replaced the X_a by their vacuum expectation values according to Eq. (31). The integral projects on the $\theta^2\bar{\theta}^2$ component of the integrand, which is

$$\mathcal{L}_{\text{mass}} = \sum_i \left(\mathcal{Z}_i\Big|_0 |F_i|^2 + \frac{\partial \mathcal{Z}_i}{\partial X_a}\Big|_0 F_a \overline{F}_i \varphi_i + \text{h.c.} + \frac{\partial^2 \mathcal{Z}_i}{\partial X_a \partial \overline{X}_b}\Big|_0 F_a \overline{F}_b\, |\varphi_i|^2\right) + \ldots \tag{36}$$

The F_a are fixed by the hidden sector dynamics according to Eq. (31), while the F_i should be eliminated using their equations of motion. These are

$$\begin{aligned} 0 = \frac{\partial \mathcal{L}}{\partial F_i} &= \frac{\partial W}{\partial \Phi_i}\Big|_0 + \mathcal{Z}_i|_0\, \overline{F}_i + \frac{\partial \mathcal{Z}_i}{\partial X_a}\Big|_0 \overline{F}_a \overline{\varphi}_i \\ \Rightarrow\ F_i &= -\frac{1}{\mathcal{Z}_i}\Big|_0 \left(\frac{\partial \overline{W}}{\partial \overline{\Phi}_i}\Big|_0 + \frac{\partial \mathcal{Z}_i}{\partial \overline{X}_a}\Big|_0 F_a \varphi_i\right)\end{aligned} \tag{37}$$

By resubstituting this into the Lagrangian Eq. (36), and collecting the F_a-dependent coefficients of $|\varphi_i|^2$ (everything that vanishes as $F_a \to 0$ must be supersymmetric), one obtains the soft SUSY-breaking scalar masses:

$$\widetilde{m}_i^2 = \left(\frac{1}{\mathcal{Z}_i} \frac{\partial \mathcal{Z}_i}{\partial X_a} \frac{\partial \mathcal{Z}_i}{\partial \overline{X}_b}\bigg|_0 - \frac{\partial^2 \mathcal{Z}_i}{\partial X_a \partial \overline{X}_b}\bigg|_0 \right) F_a \overline{F}_b \qquad \text{(no sum over } i) \,. \tag{38}$$

In this field basis $\mathcal{Z}_i|_0$ multiplies also the kinetic terms $\partial_\mu \overline{\varphi}_i \partial^\mu \varphi_i$ in the visible sector. To obtain the scalar soft masses for canonically normalised fields, we should rescale $\varphi_i \to \varphi_i / \sqrt{\mathcal{Z}_i|_0}$, which finally gives

$$\boxed{\begin{aligned} m_i^2 &= \left(\frac{1}{\mathcal{Z}_i^2} \frac{\partial \mathcal{Z}_i}{\partial X_a} \frac{\partial \mathcal{Z}_i}{\partial \overline{X}_b}\bigg|_0 - \frac{1}{\mathcal{Z}_i} \frac{\partial^2 \mathcal{Z}_i}{\partial X_a \partial \overline{X}_b}\bigg|_0 \right) F_a \overline{F}_b \\ &= - \frac{\partial}{\partial X_a} \frac{\partial}{\partial \overline{X}_b} \log \mathcal{Z}_i \bigg|_0 F_a \overline{F}_b \,. \end{aligned}} \tag{39}$$

For instance, in a toy model of a single visible sector field Φ, a single hidden sector field $\langle X \rangle = F\theta^2$, and an effective Kähler potential

$$K = \left(1 - \frac{|X|^2}{M^2} \right) |\Phi|^2 \,, \tag{40}$$

the scalar soft mass is

$$m^2 = \frac{|F|^2}{M^2} \,. \tag{41}$$

4.2 Gaugino masses

Just as scalar soft masses are encoded in the X_a-dependence of the Kähler potential K, gaugino masses are encoded in the X_a-dependence of the gauge kinetic function S. For each of the MSSM gauge factors, the gauge kinetic Lagrangian is

$$\mathcal{L}_{\text{gk}} = \frac{1}{4} \int \mathrm{d}^2\theta \; S\left(\frac{X^a}{M} \right) W^\alpha W_\alpha + \text{h.c.} \tag{42}$$

If S has a non-vanishing θ^2 component in the vacuum (because the X_a have non-vanishing θ^2 components), then the gaugino mass is nonzero since

$$\operatorname{tr} W^\alpha W_\alpha = \operatorname{tr} \lambda\lambda + \ldots \tag{43}$$

and the integral in Eq. (42) again projects out the θ^2 of the integrand. More precisely,

$$\mathcal{L}_{\text{gk}} = S\Big|_0 \left(-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + i \operatorname{tr} \lambda \sigma^\mu \partial_\mu \overline{\lambda} \right) + \frac{1}{4} \frac{\partial S}{\partial X_a}\bigg|_0 F^a \operatorname{tr} \lambda\lambda + \text{h.c.} + \ldots \tag{44}$$

To obtain canonically normalised fields, λ should be rescaled as $\lambda \to \lambda / \sqrt{S|_0}$ (assuming $S|_0$ is real, i.e. no Θ terms), which gives the gaugino mass in the canonical basis:

$$\boxed{M_\lambda = -\frac{1}{2} \frac{1}{S} \frac{\partial S}{\partial X_a}\bigg|_0 F^a = -\frac{1}{2} \frac{\partial \log S}{\partial X_a}\bigg|_0 F^a \,.} \tag{45}$$

For example, in a toy model with

$$\mathcal{L}_{\rm gk} = \frac{1}{4}\int \mathrm{d}^2\theta \, \left(1 + \frac{X}{M}\right) \mathrm{tr}\, W^\alpha W_\alpha + \text{h.c.} \tag{46}$$

and $\langle X \rangle = F\theta^2$, we would get $M_\lambda = -F/(2M)$.

All of this can easily be generalised to non-abelian gauge theories, and the result for the gaugino mass is unchanged.

4.3 All the rest

The higgsino mass parameter μ plays a special role among the dimensionful MSSM parameters, because it is supersymmetric. A priori there is no reason why its size should be connected to the scale of SUSY breaking; while the typical size of the other soft terms is F_a/M as we have seen, supersymmetry allows for an arbitrary μ parameter in the superpotential,

$$W = \widehat{\mu}\, H_u H_d + \ldots \tag{47}$$

where H_u and H_d are now to be regarded as superfields. However, μ also receives contributions from SUSY breaking by what is often called the *Giudice-Masiero mechanism.* More precisely, a similar procedure as the one we just carried out for gauginos and scalars gives a canonically normalised effective μ parameter

$$\boxed{\mu = \frac{1}{\left(\mathcal{Z}_{H_u}\mathcal{Z}_{H_d}|_0\right)^{1/2}} \left(\widehat{\mu} - \frac{\partial}{\partial \overline{X}_a}\mathcal{H}_{H_u H_d}\Big|_0 \overline{F}_a\right)\,,} \tag{48}$$

where $\mathcal{H}$ is a function of the hidden sector superfields as in Eq. (33). If $\widehat{\mu}$ is forced to be zero by some symmetry, what remains is an effective μ parameter which is of the order of the soft breaking terms.

Finally, the trilinear a and bilinear b soft terms can be worked out by the same methods, but the results are somewhat more complicated because they depend on the $\mathcal{Z}_i$, the $\mathcal{H}_{ij}$, and also on possible higher-dimensional operators in the superpotential.

4.4 Generalisation to supergravity

Using the formalism of Section 3.3, Eqns. (39), (45) and (48) can be generalised to supergravity. For simplicity we restrict ourselves to vacua with vanishing cosmological constant. It turns out that the gaugino mass formula Eq. (45) does not change (there is no ϕ dependence in the gauge kinetic action), while the scalar soft masses gets an extra contribution given by the gravitino mass $m_{3/2}$,

$$m_i^2 = m_{3/2}^2 - \frac{\partial}{\partial X_a}\frac{\partial}{\partial \overline{X}_b}\log \mathcal{Z}_i\Big|_0 F_a \overline{F}_b\,. \tag{49}$$

The μ parameter also changes:

$$\mu = \frac{1}{\left(\mathcal{Z}_{H_u}\mathcal{Z}_{H_d}|_0\right)^{1/2}} \left(m_{3/2}\mathcal{H}_{H_u H_d} + e^{K_{\rm hid,}/2M_P^2}\widehat{\mu} - \frac{\partial}{\partial \overline{X}_a}\mathcal{H}_{H_u H_d}\Big|_0 \overline{F}_a\right)\,. \tag{50}$$

The canoncially normalised gravitino mass is related to the compensator (gravitational) F-term and the chiral hidden sector F-terms as

$$m_{3/2} = F_\phi + \frac{1}{3M_P^2}\, \frac{\partial}{\partial X^a} K \bigg|_0 F^a \,. \tag{51}$$

In models where SUSY is broken in the rigid limit, the second term is typically negligible because all hidden-sector fields with Planck-scale VEVs have been integrated out. However, this is in general not true in models where the hidden sector involves gravitational degrees of freedom. Often the former is assumed in gauge-mediated models (where the messenger scale M of SUSY breaking mediation is much lower than M_P), and the latter in gravity-mediated models (where the messenger scale is identified with M_P). Hence, in gauge-mediated models, the gravitino mass is negligible with respect to the typical soft mass scale,

$$m_{3/2} \sim F_\phi \sim \frac{F_a}{M_P} \qquad \ll \qquad m_{\rm soft} \sim \frac{F_a}{M}\,, \tag{52}$$

while in generic gravity-mediated models $m_{3/2}$ and $m_{\rm soft}$ are comparable.

A toy example for gravity mediation is given by the simplest "no-scale" model, defined by

$$K = -3\, M_P^2 \log\left(\frac{X + \overline{X}}{M_P} + \frac{|\Phi|^2}{M_P^2}\right) \qquad W = \text{const.}\,, \tag{53}$$

which is easily seen to lead to $F_\phi = 0$ and a scalar soft mass

$$m_\Phi^2 = 0 \tag{54}$$

because the two terms in Eq. (49) precisely cancel each other.