

Determination of Parton Densities in Deep Inelastic Scattering

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H1 Collaboration, DESY





Overview

- **Electron-Proton scattering and the Quark Parton Model**
- **Improving the QPM: QCD and Evolution Equations**
- **Approaches for Evolving Parton Distributions**
- **Global Structure Function Data**
- **Extracting Parton Distributions**



Electron-Proton scattering and the Quark Parton Model

Quark Parton Model:

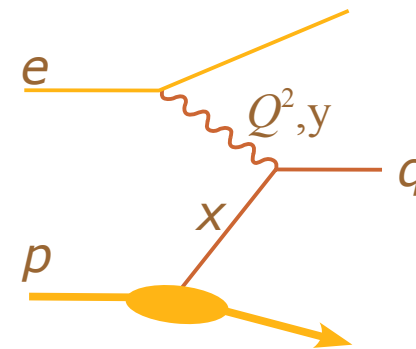
- ep - scattering \rightarrow eq - scattering via photon exchange with

x virtuality Q^2

x inelasticity y , in proton rest frame:

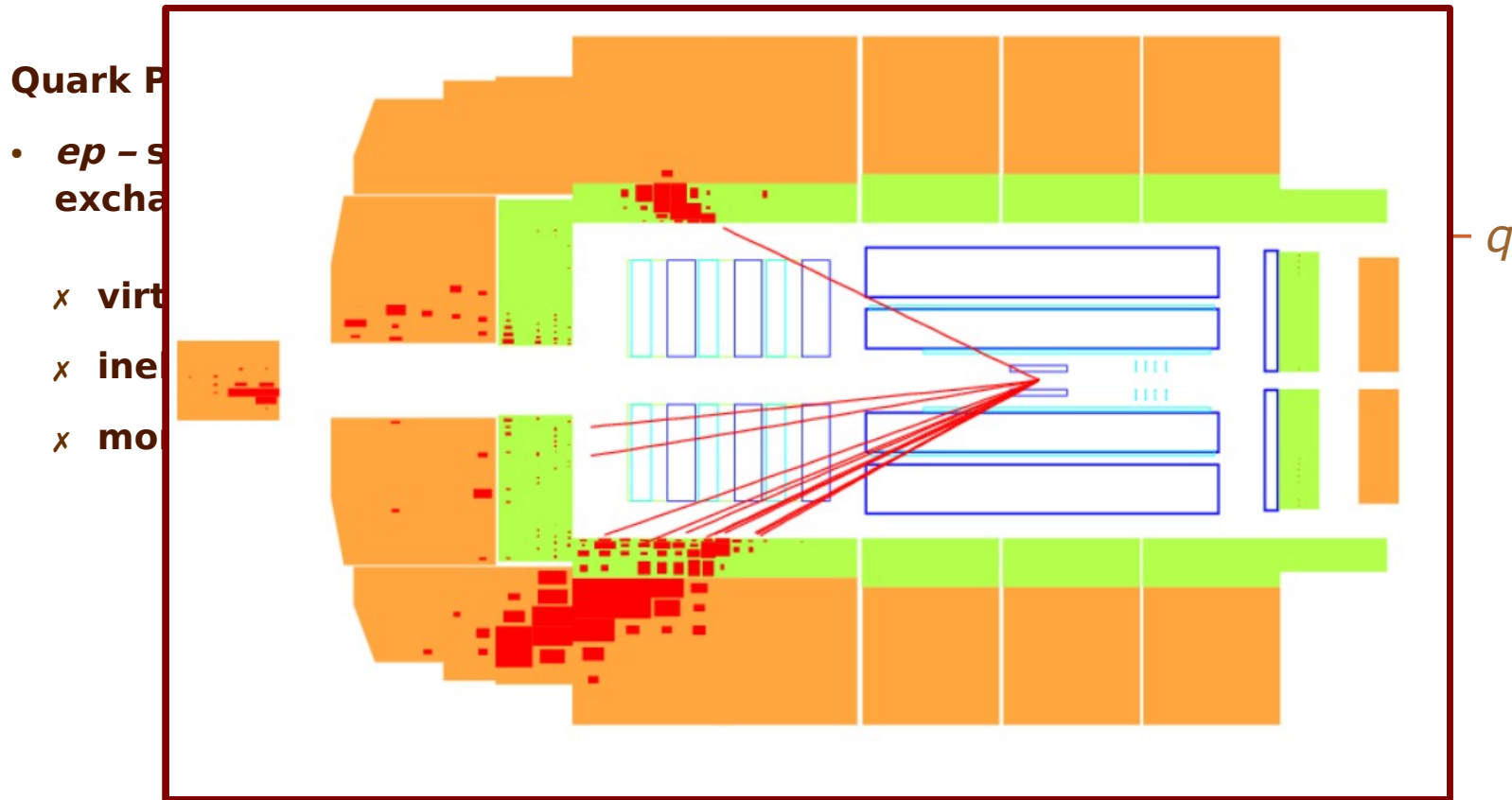
x momentum fraction $x = \frac{Q^2}{y \cdot E_{cms}^2}$

$$\frac{E_y^{prf}}{E_e^{prf}}$$





Electron-Proton scattering and the Quark Parton Model





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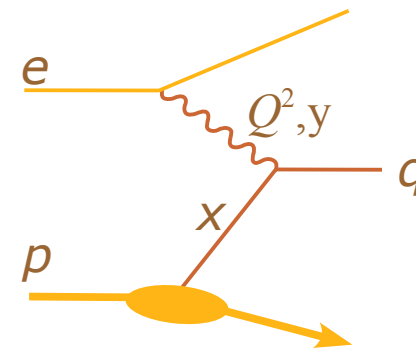
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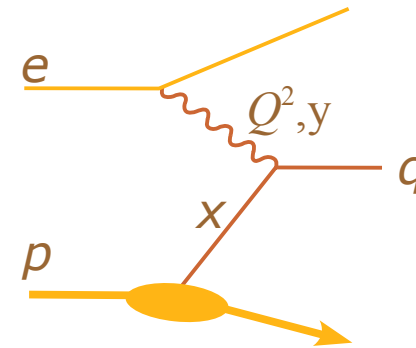
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$\rightarrow q_i(x)$: parton distribution function of flavour i





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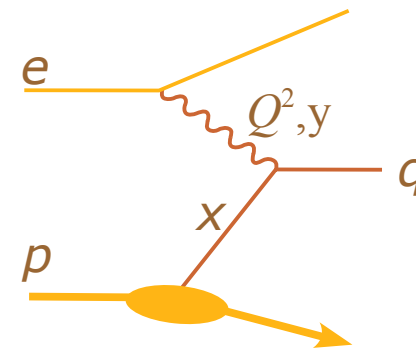
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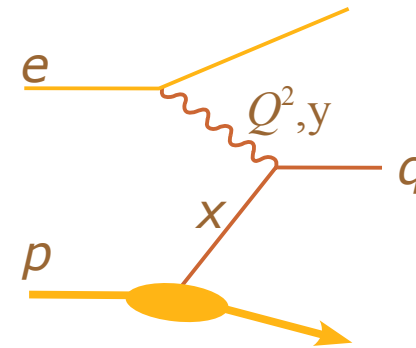
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\rightarrow structure function $F_2(x, Q^2)$

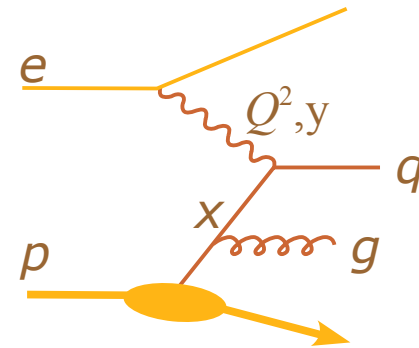


Quark Parton Model

Improving the QPM: QCD and Parton Evolution

QCD Improved Quark Parton Model

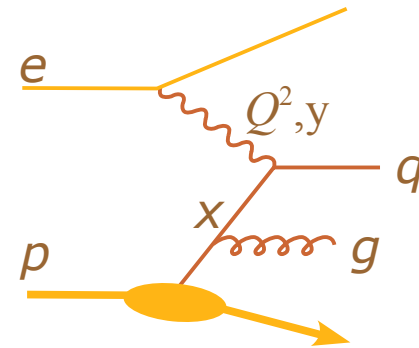
- include perturbative QCD



Improving the QPM: QCD and Parton Evolution

QCD Improved Quark Parton Model

- include perturbative QCD via
 x initial state gluon radiation

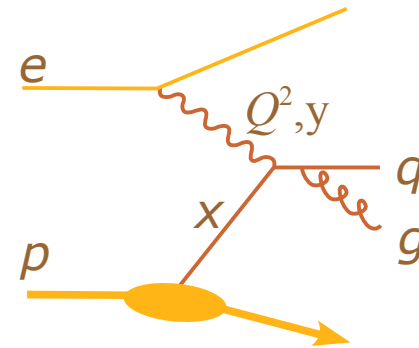




Improving the QPM: QCD and Parton Evolution

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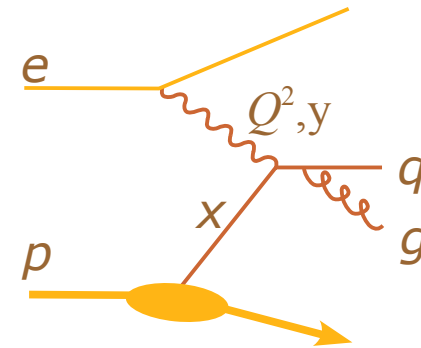




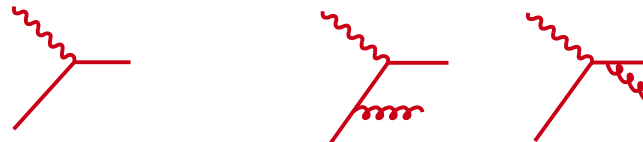
Improving the QPM: QCD and Parton Evolution

QCD Improved Quark Parton Model

- include perturbative QCD via
 - x initial state gluon radiation
 - x final state gluon radiation



$$F_2(x, Q^2) = x \cdot e_i^2 \cdot \int_x^1 \frac{d\xi}{\xi} q_i(\xi) \left[\delta\left(1 - \frac{x}{\xi}\right) + \frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{\xi}\right) \ln\left(\frac{Q^2}{\mu^2}\right) \right]$$



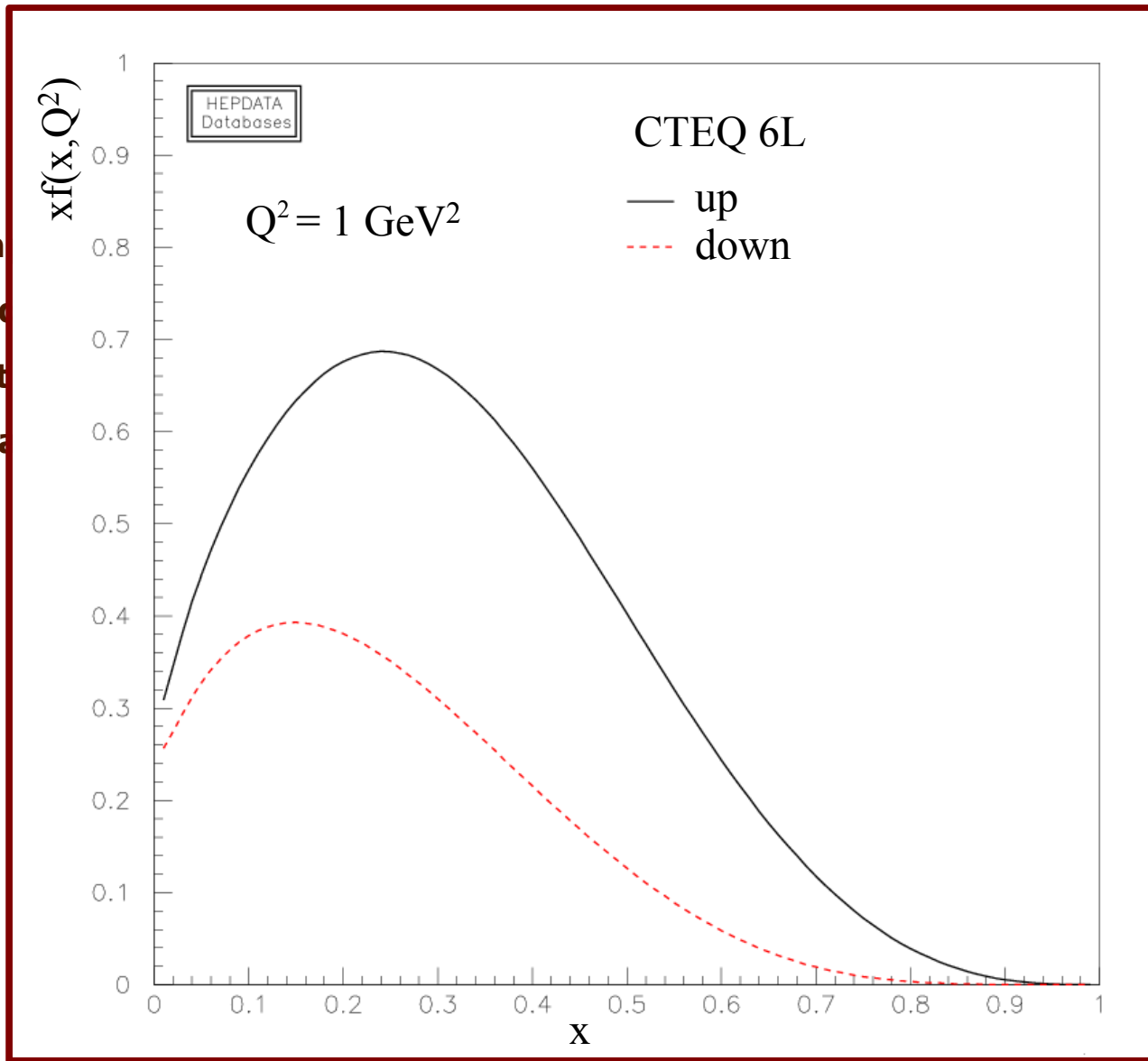


QCD Im

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x init

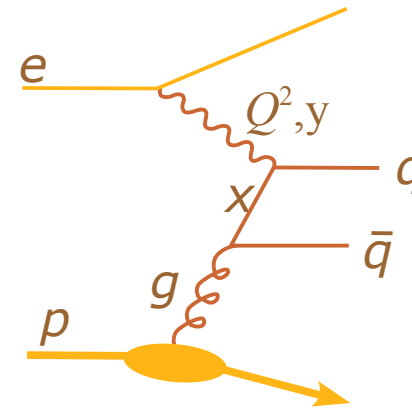
x fina



Improving the QPM: QCD and Parton Evolution

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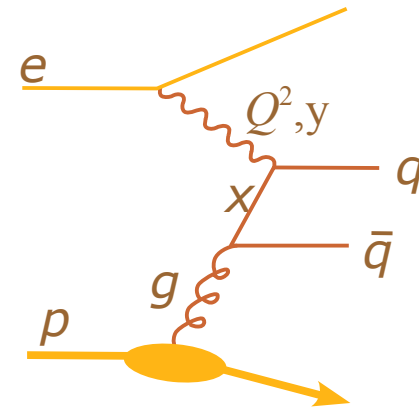
- include perturbative QCD via
 - x initial state gluon radiation
 - x final state gluon radiation
 - x gluon splitting
 - **Boson Gluon Fusion**



Improving the QPM: QCD and Parton Evolution

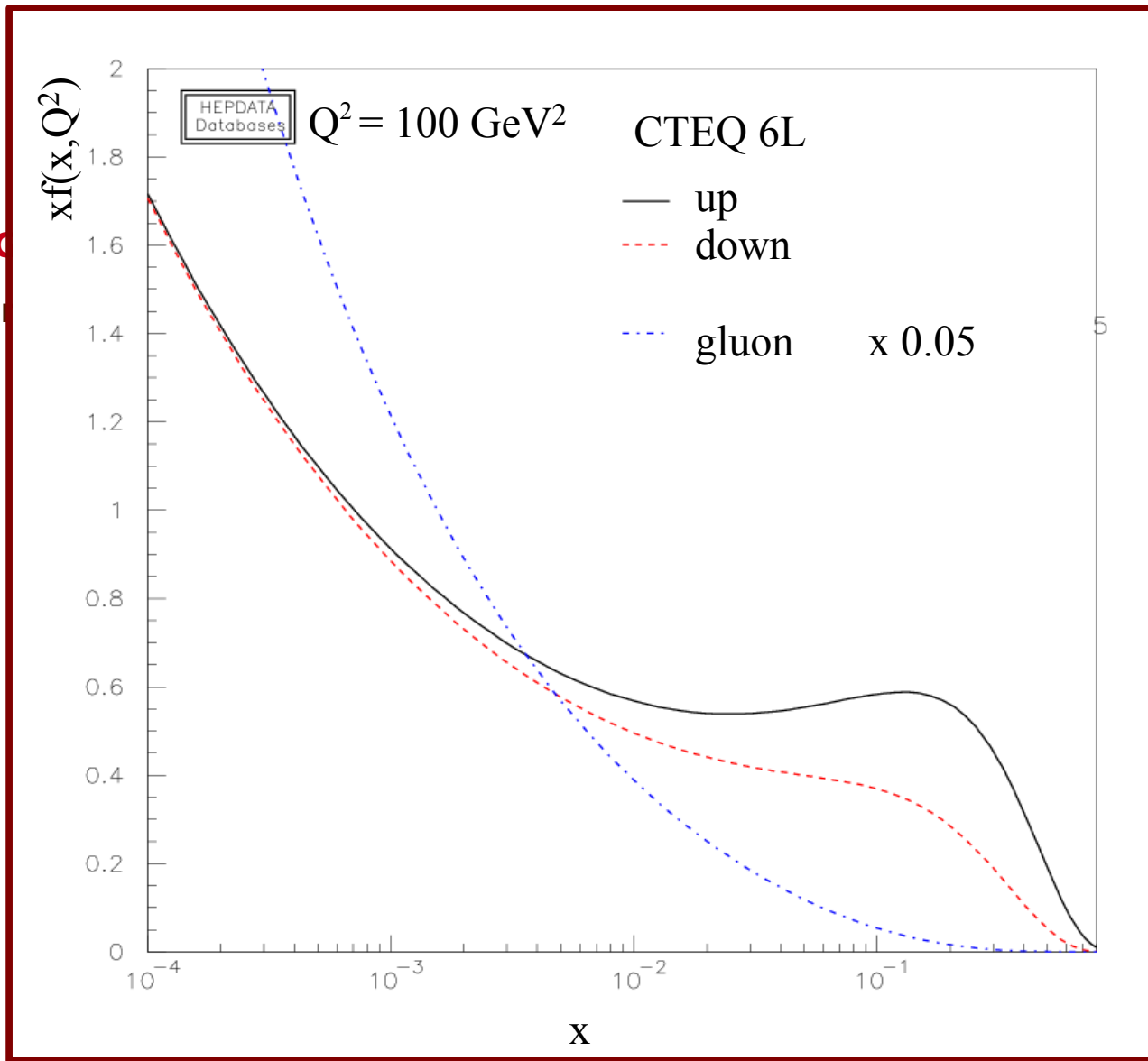
Boson Gluon Fusion

- dominant process at low x





Boson C
 • domin

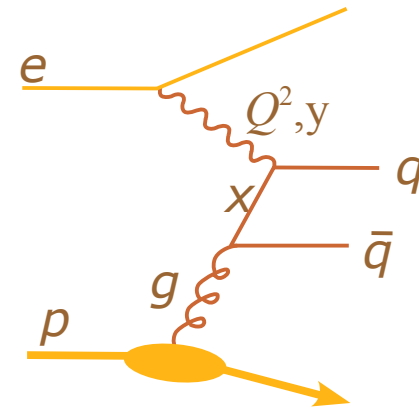


n
 Q^2, y
 q
 \bar{q}

Improving the QPM: QCD and Parton Evolution

Boson Gluon Fusion

- dominant process at low x
- $p\bar{p}$ scattering at low x dominated by
→ **Gluon Gluon Fusion**

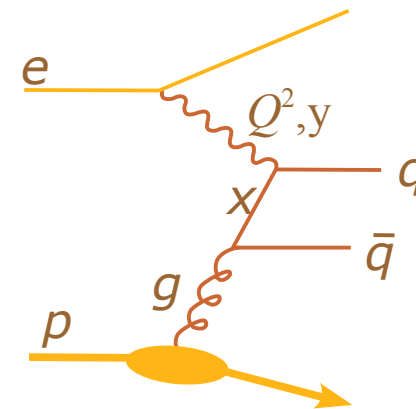
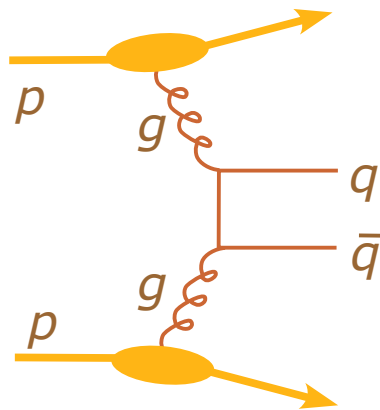




Improving the QPM: QCD and Parton Evolution

Boson Gluon Fusion

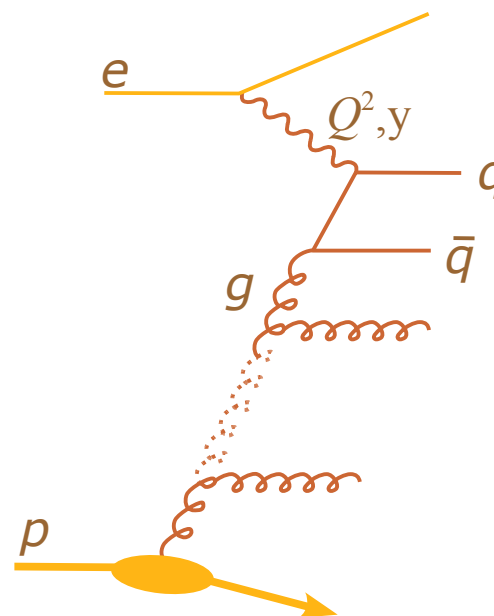
- dominant process at low x
- $p\bar{p}$ scattering at low x dominated by
 → **Boson Gluon Fusion**



→ HERA's impact on
Tevatron and LHC

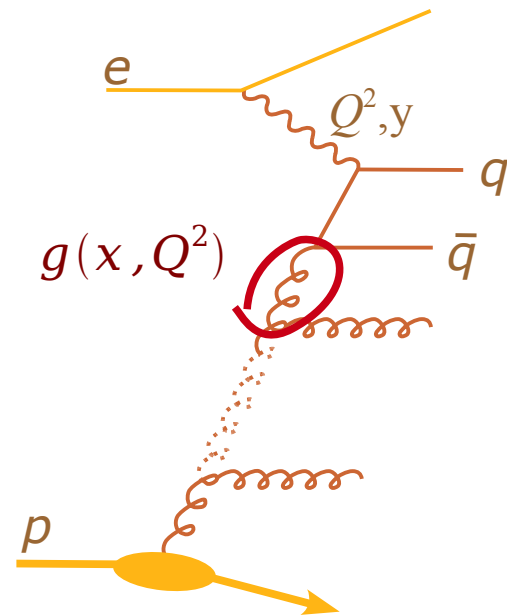


Improving the QPM: QCD and Parton Evolution





Improving the QPM: QCD and Parton Evolution



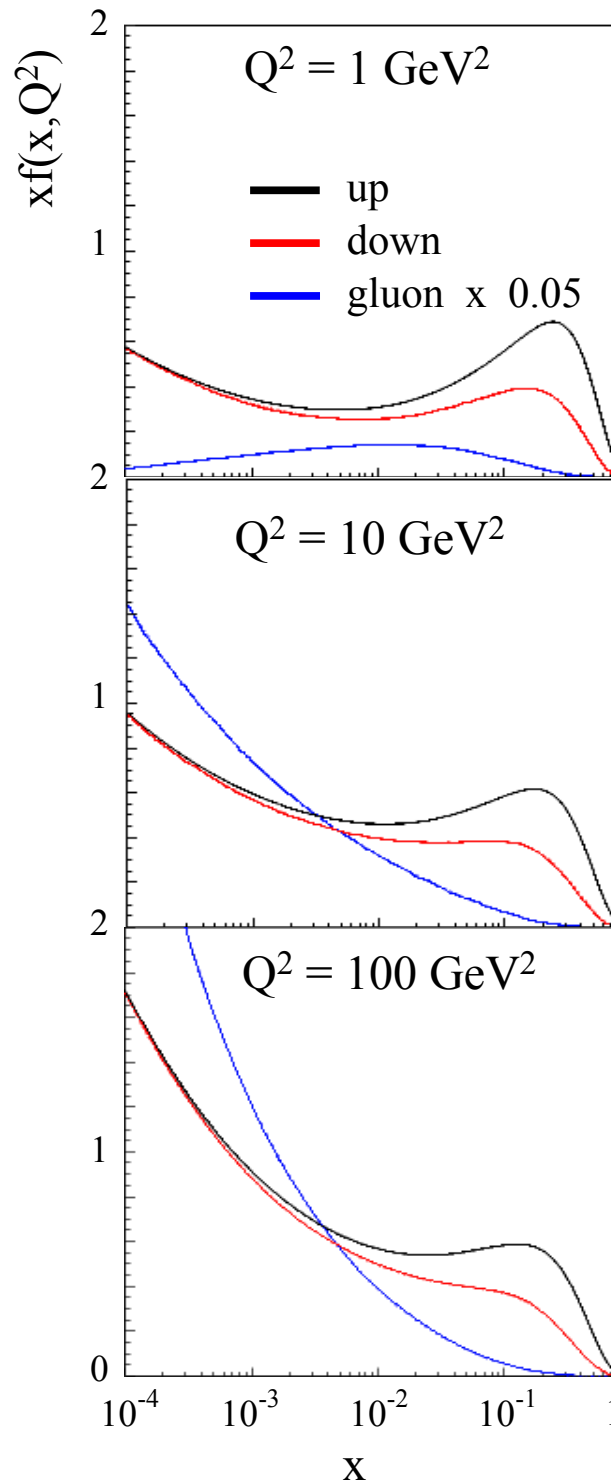
Improving t

Parton Evolution Equation

- calculate PDF $f_i(x, Q^2)$ a
- after parton radiation
- gluon evolution:

$$\frac{\partial g(x, \mu^2)}{\partial \ln \mu^2} \approx \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} P_{gg}(\xi) g(\xi, \mu^2)$$

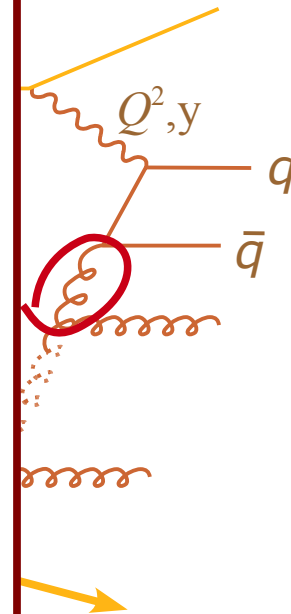
• $P_{gg}(x/\xi)$: $g \rightarrow gg$ spl



in DIS



olution

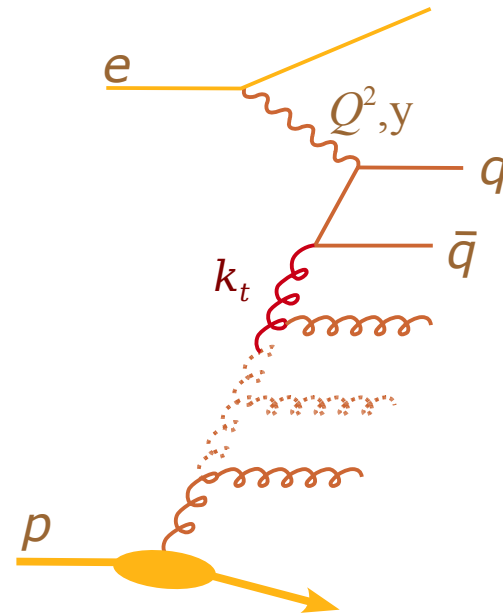




Approaches for Evolving Parton Distributions

Collinear Approximation:

- transverse momentum of **incoming** gluon negligible: $k_t \ll Q^2 \Rightarrow k_t \approx 0$

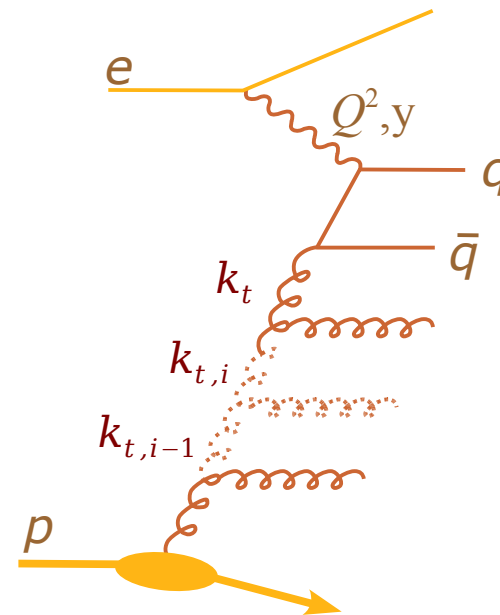




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- in each splitting $k_{t,i-1} \ll k_{t,i}$ k_t ordering

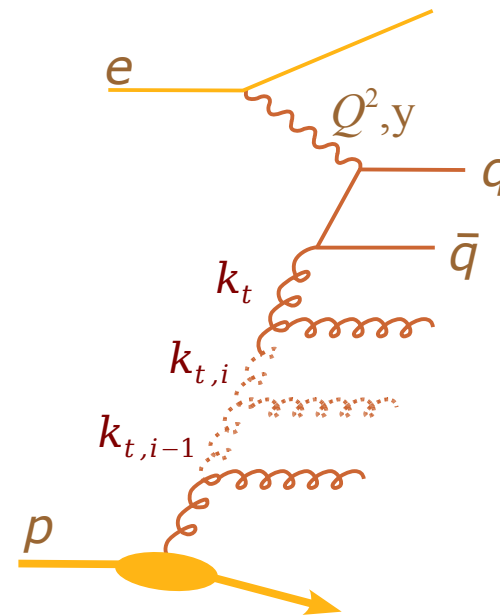




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- in MC event generators: only $k_{t,i-1} < k_{t,i}$ realizable
- subsequent adjustment of kinematics necessary

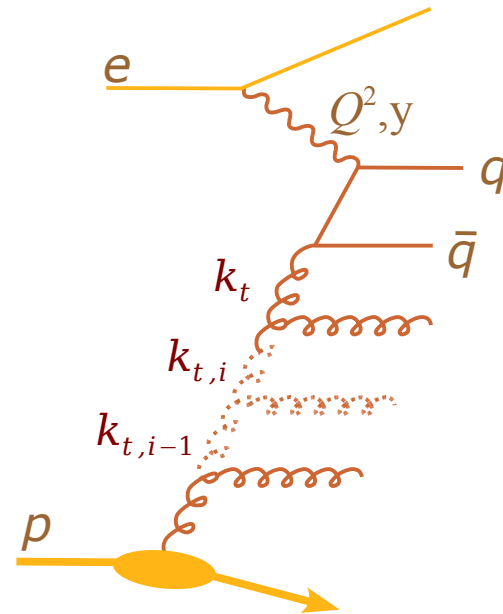




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k_t factorisation:

- transverse momentum of incoming gluon fully taking into account: $k_t \neq 0$

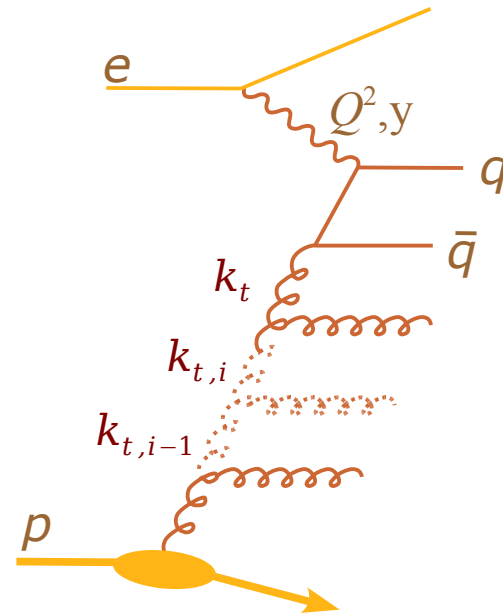




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- PDFs k_t dependent
 → **unintegrated** PDFs $A(x, k_t, \bar{\mu})$

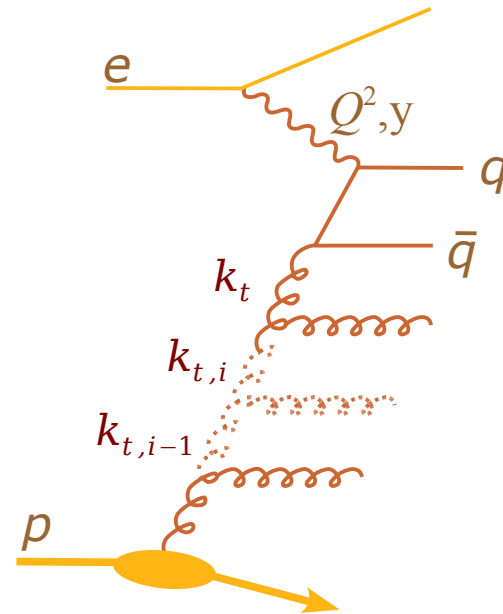




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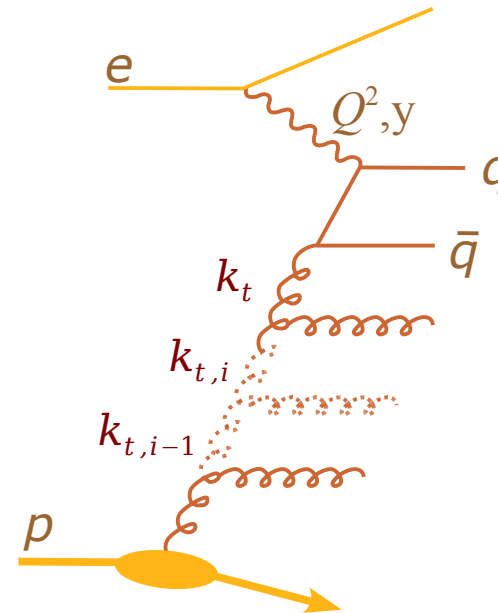
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- CCFM evolution equations
 - x fulfill k_t ordering at high x



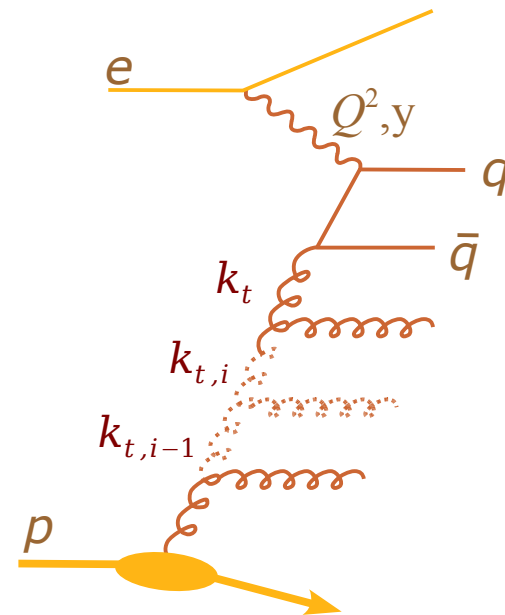


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- CCFM evolution equations
 - x fulfill k_t ordering at high x
- implemented in MC event generator

CASCADE

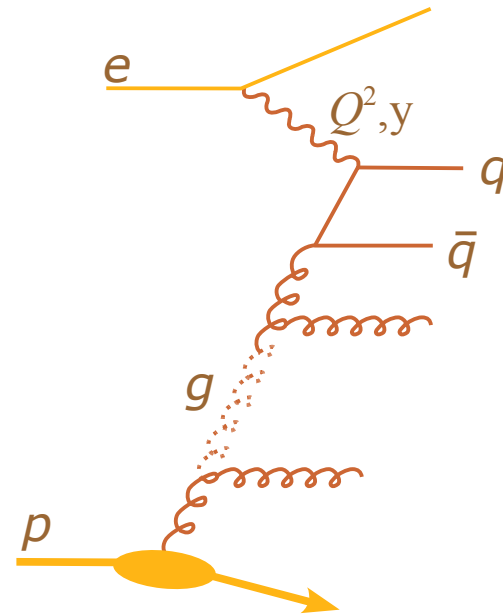


Global Structure Function Data

Data suited for PDF extraction

- Structure Function Data from various experiments:

$x F_2^{ep}$: H1, ZEUS, BCDMS, NMC, ...



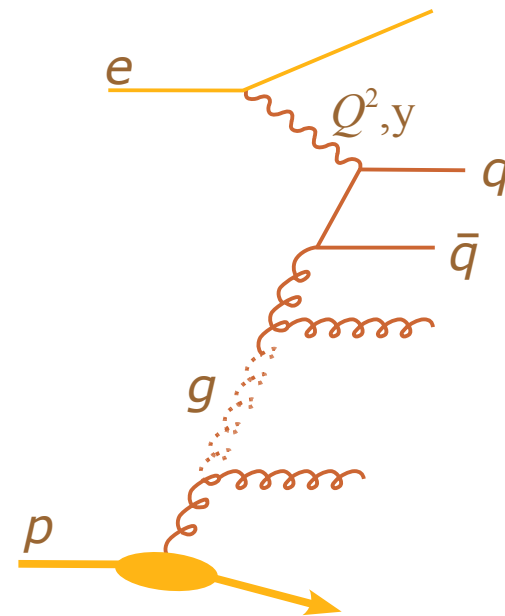
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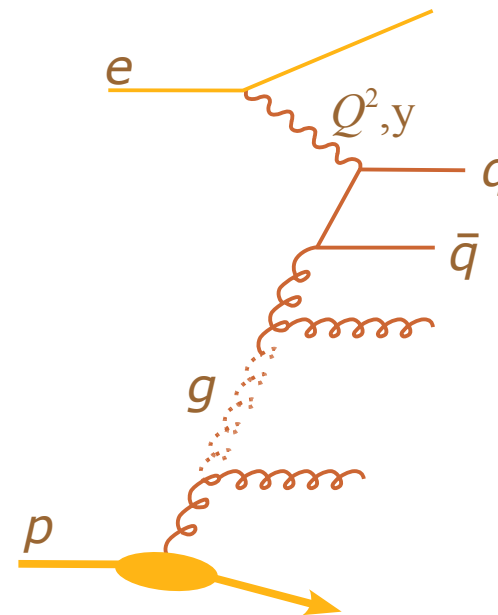
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x $F_2^{\mu d}/F_2^{\mu p}$: NMC

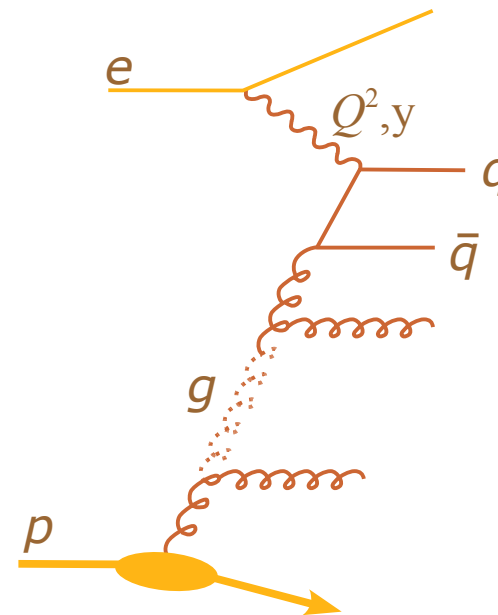
→ tests ratio u_v/d_v



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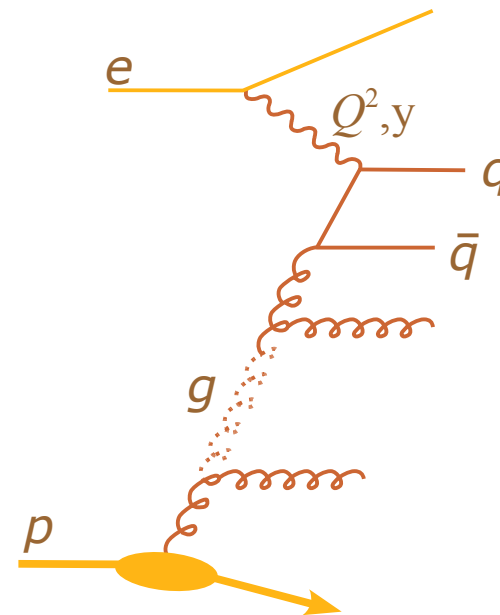
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 - x $F_3^{\nu, \bar{\nu}}$: parity violating structure function from (anti-) neutrino scattering at CCFR



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 - x $F_2^{c\bar{c}}$: ZEUS, H1
 - x ...





Global

Data suited for P

- Structure Functions experiments:

x F_2^{ep} : H1, ZEUS

x F_L : SLAC, BCDMS

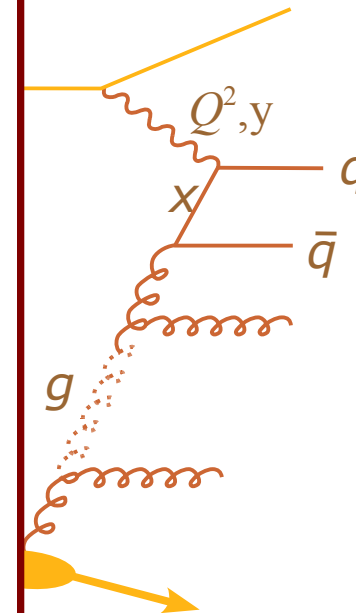
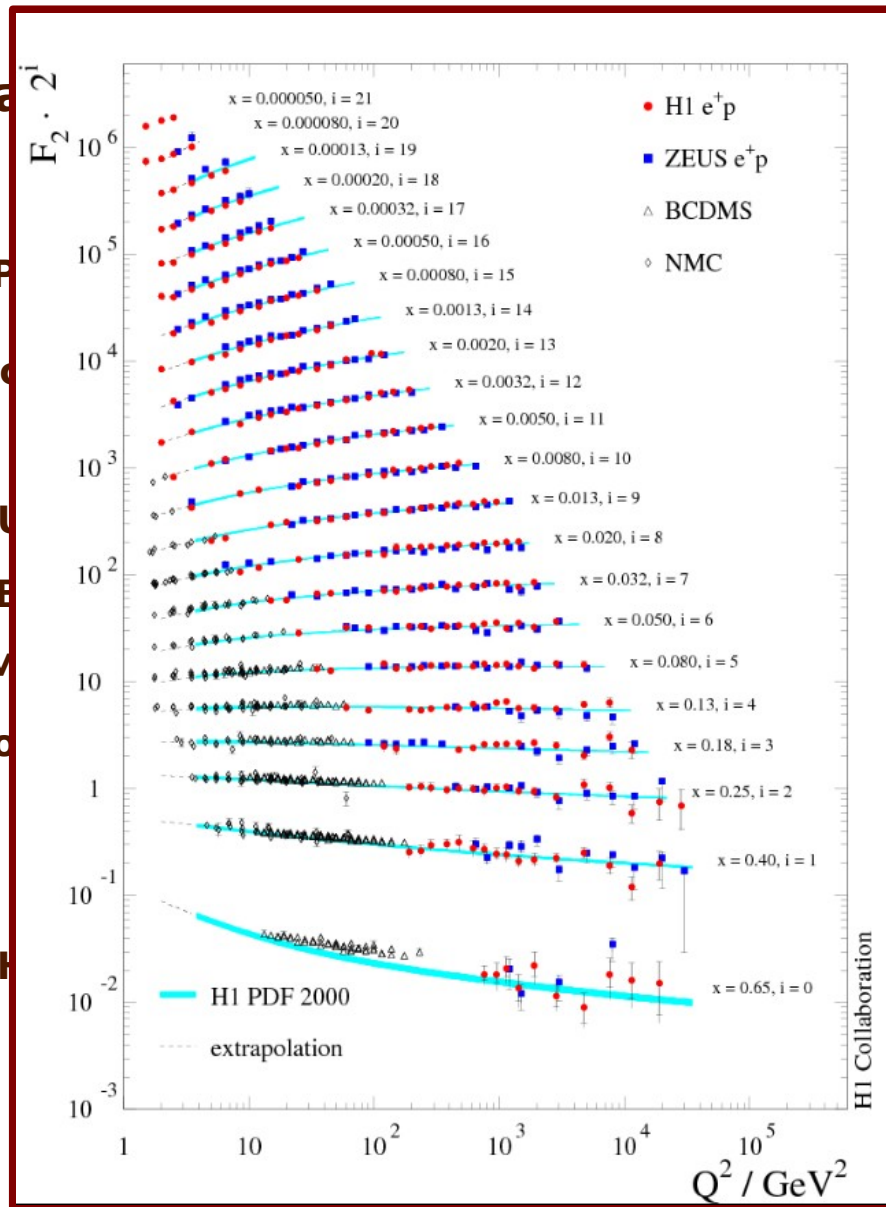
x $F_2^{\mu d} / F_2^{\mu p}$: NMC

→ tests ratios

x $F_3^{v, \bar{v}}$: parity from (anti-)

x $F_2^{c\bar{c}}$: ZEUS, H1

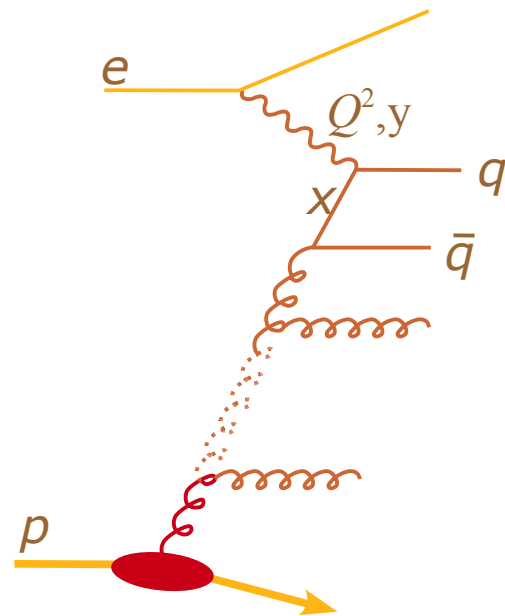
x ...





Extracting Parton Distributions

Composition of the Proton





Extracting Parton Distributions

Composition of the Proton

- choose parametrisation of **initial PDFs**:

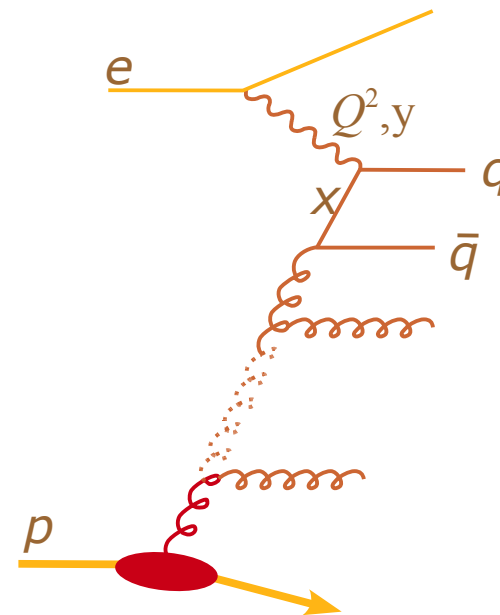
$$u(x) = N_u x^{\alpha_u} (1-x)^{\beta_u} P_u(x)$$

$$d(x) = N_d x^{\alpha_d} (1-x)^{\beta_d} P_d(x)$$

$$S(x) = N_S x^{\alpha_S} (1-x)^{\beta_S} P_S(x)$$

$$g(x) = N_g x^{\alpha_g} (1-x)^{\beta_g} P_g(x)$$

x $P_i(x)$ polynomial in x or \sqrt{x}





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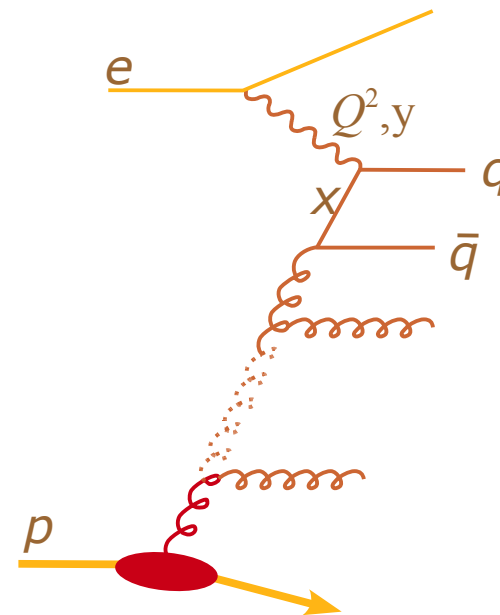
x $P_i(x)$ polynomial in x or \sqrt{x}

- k_t factorisation as implemented in MC

generator CASCADE:

$$A_0(x, k_t^2) = N x^\alpha (1-x)^\beta \exp\left(-\frac{(k_t - \langle k_t \rangle)^2}{\sigma^2}\right)$$

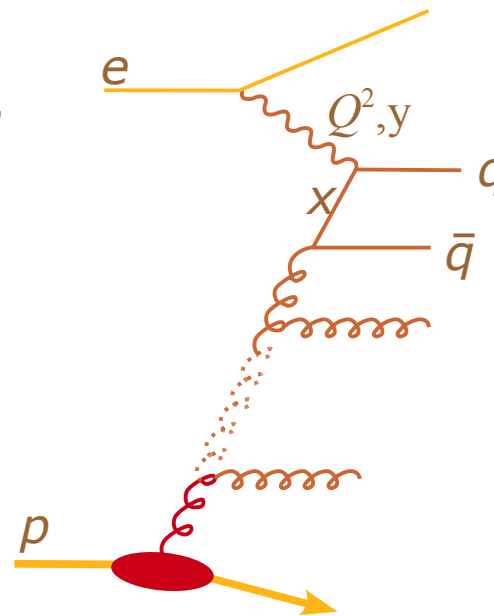
intrinsic k_t distribution



Extracting Parton Distributions

Performing χ^2 Fits to Inclusive Data:

- various methods for solving evolution equations,
e. g.
 - x QCDNUM using NLO matrix elements
 - x MC event generators

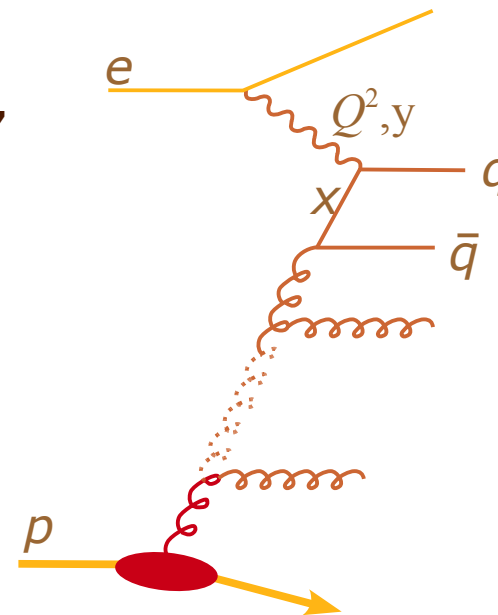


Extracting Parton Distributions

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 - × MC event generators
- PDF extraction results from **minimization of**

$$\chi^2 = \sum_i \frac{(d_i - t_i)^2}{\sigma_i^2} + \sigma_{corr}$$



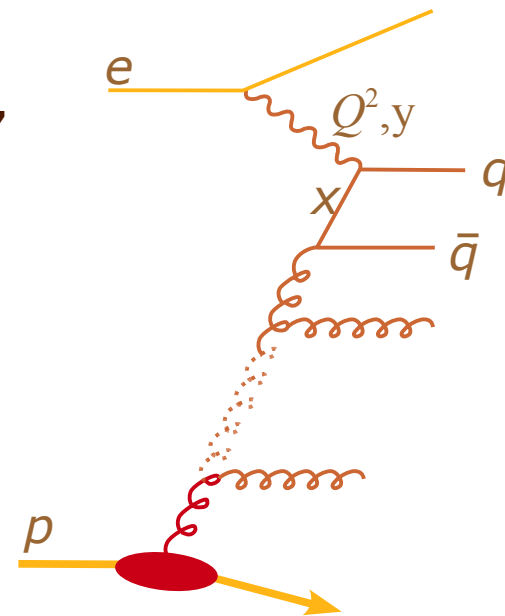
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$$\chi^2 = \sum_i \frac{(d_i - t_i)^2}{\sigma_i^2} + \sigma_{corr}$$

- estimate uncertainty on PDFs parameters by
 $\Delta\chi^2 = 1$





Extra

Performing χ^2 Fit

- various methods
- e. g.

\times QCDNUM using

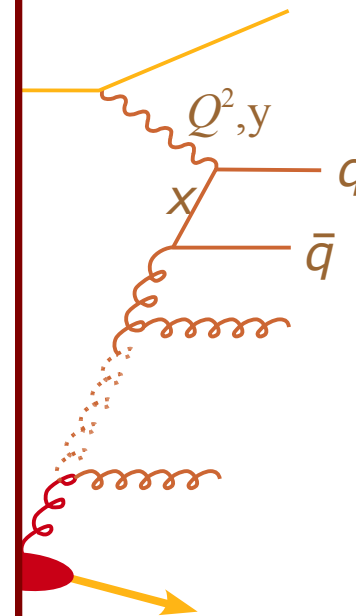
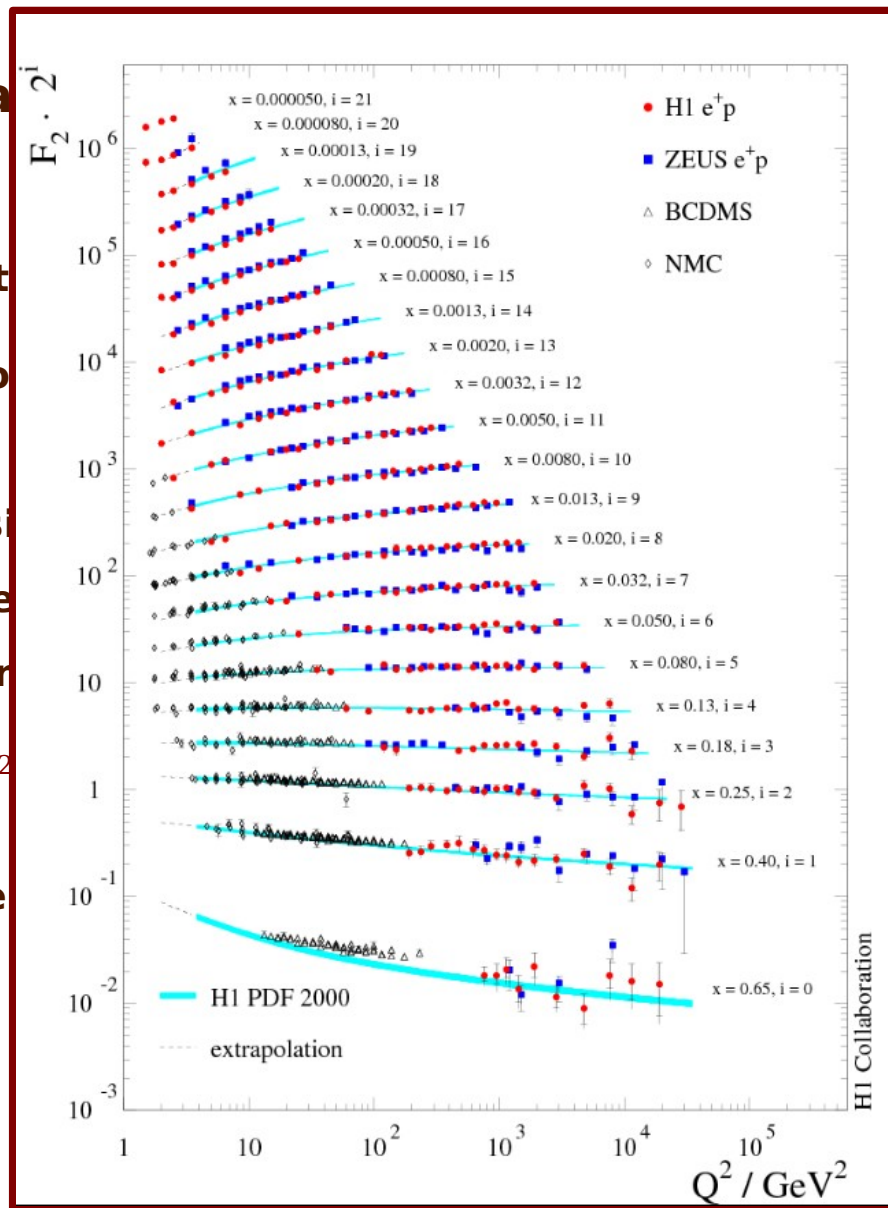
\times MC event generators

- PDF extraction

χ^2

- estimate uncertainties

$$\Delta \chi^2 = 1$$



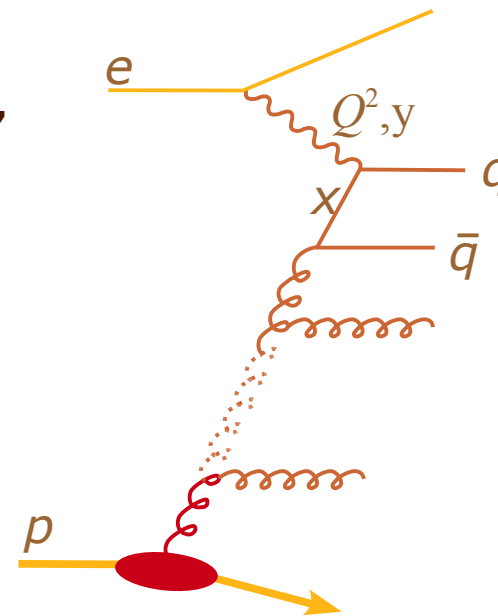
Extracting Parton Distributions

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- PDF extraction results from **minimization of**

$$\chi^2 = \sum_i \frac{(d_i - t_i)^2}{\sigma_i^2} + \sigma_{corr}$$

- estimate uncertainty on PDFs parameters by $\Delta\chi^2 = 1$
 - x current debate: uncertainties underestimated for large data sets?
 - CTEQ: $\Delta\chi^2 = 100$ better choice?





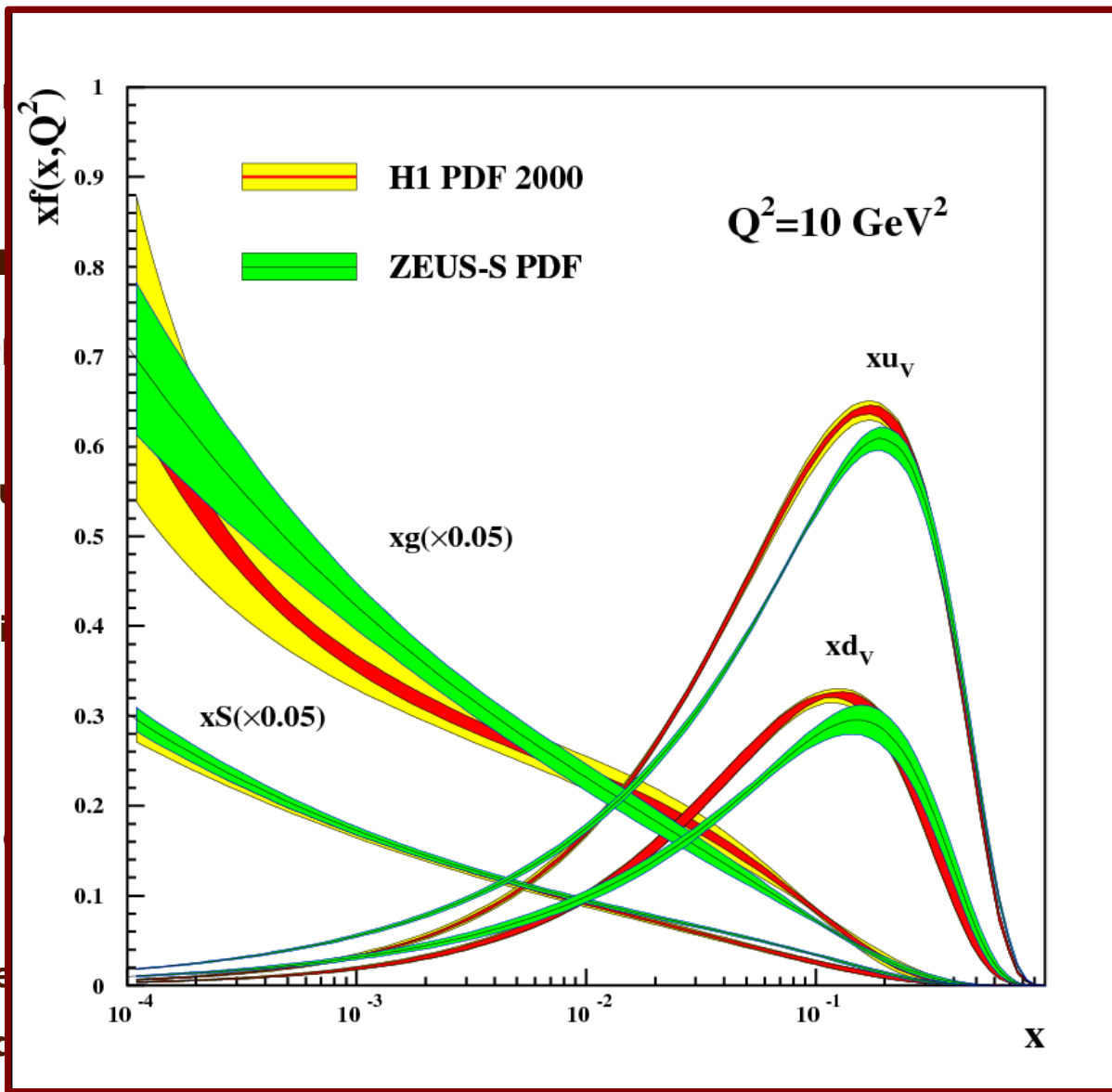
Ext

Performing χ^2

- various methods
- e. g.
- x QCDNUM
- x MC event
- PDF extraction

- estimate uncertainty
- $\Delta\chi^2=1$
- x current density
- for large Q^2

→ CTEQ: $\Delta\chi^2=100$ better choice?

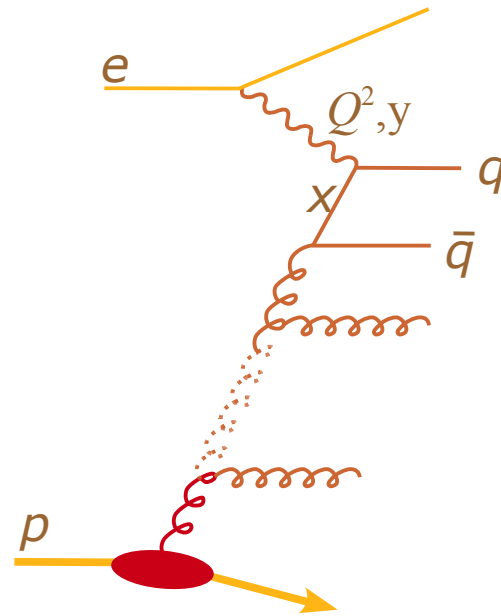


q
 \bar{q}

Extracting Parton Distributions

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e. g. CASCADE:

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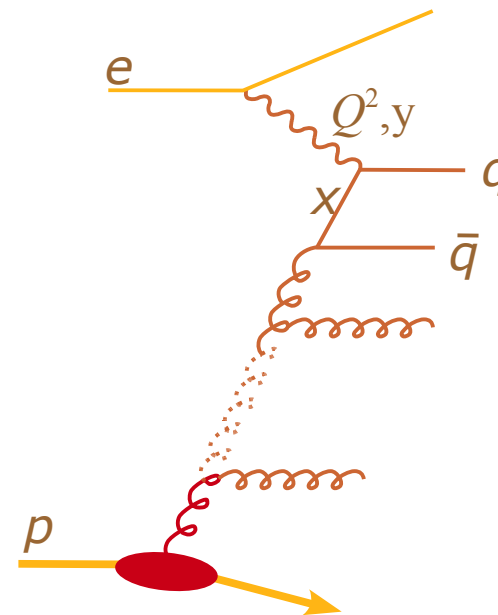
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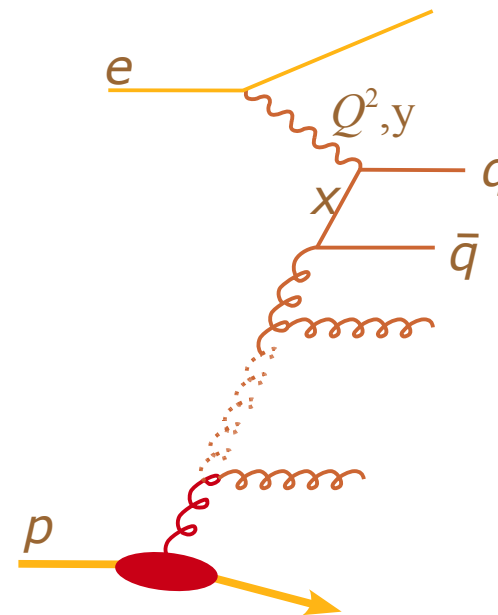
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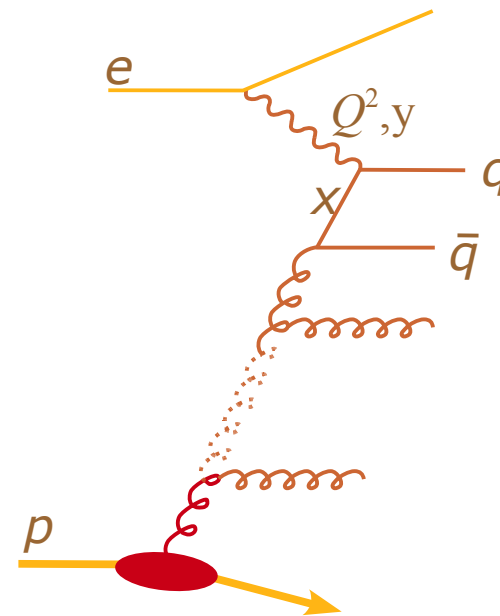
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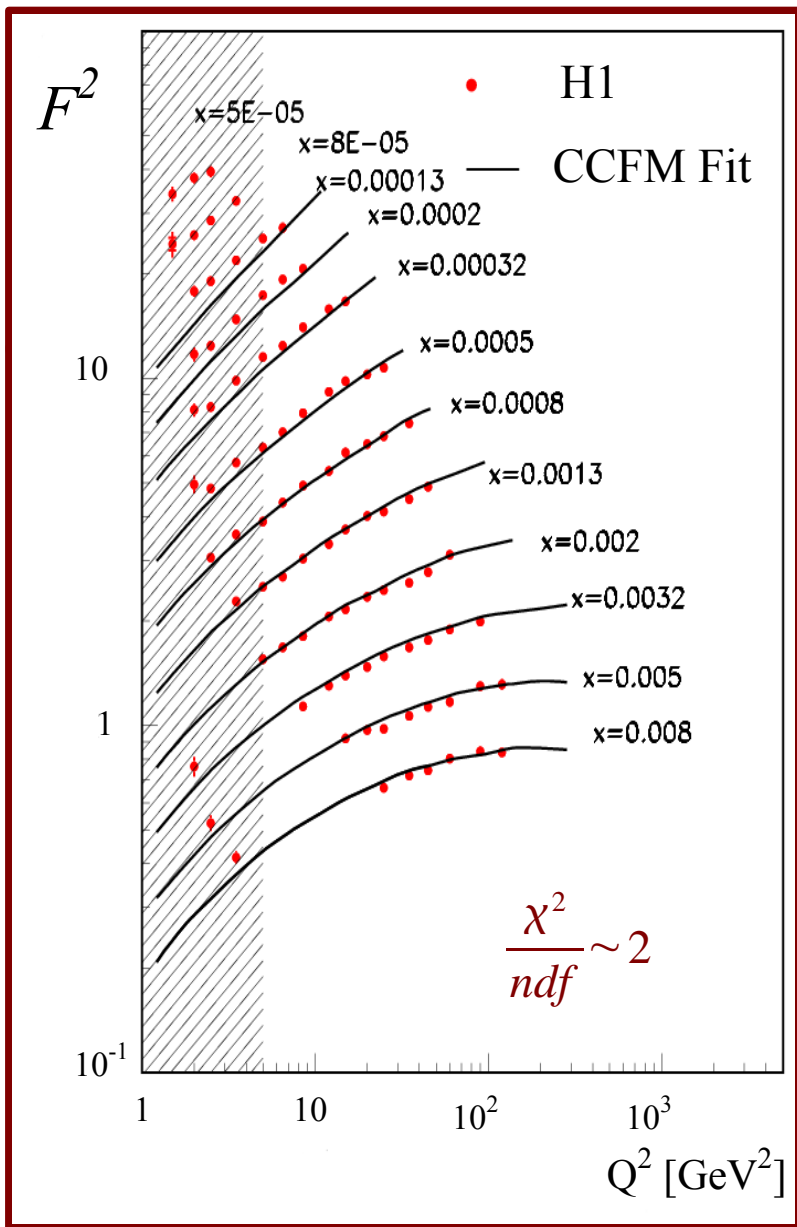
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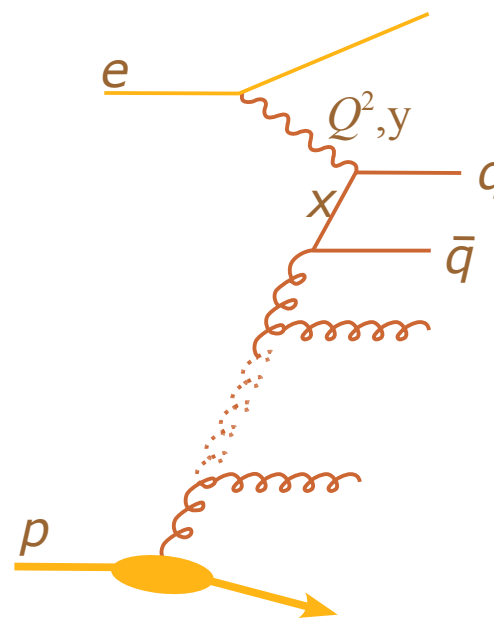
Contributions

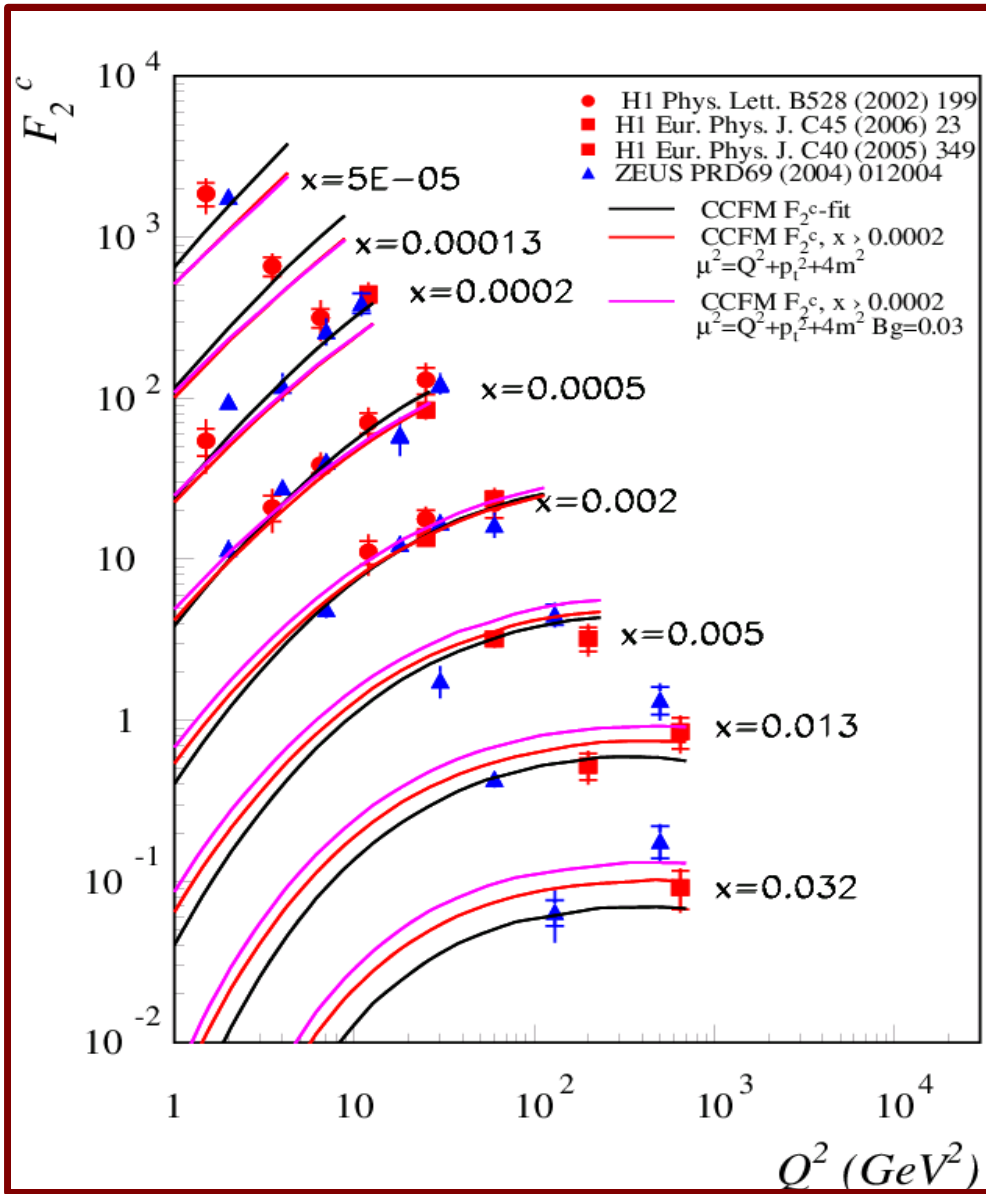
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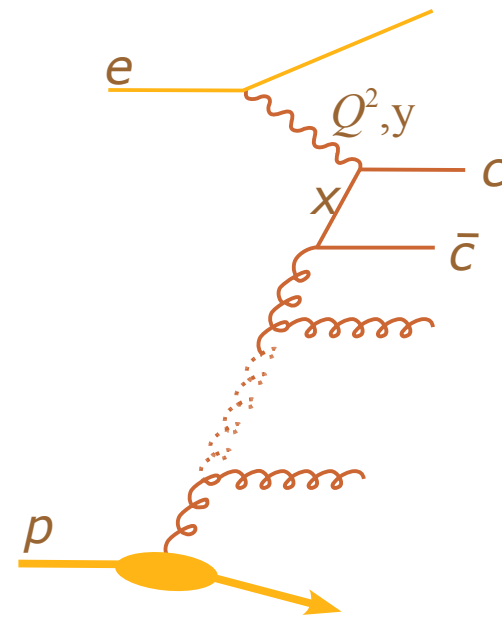
on step

, RIVET)





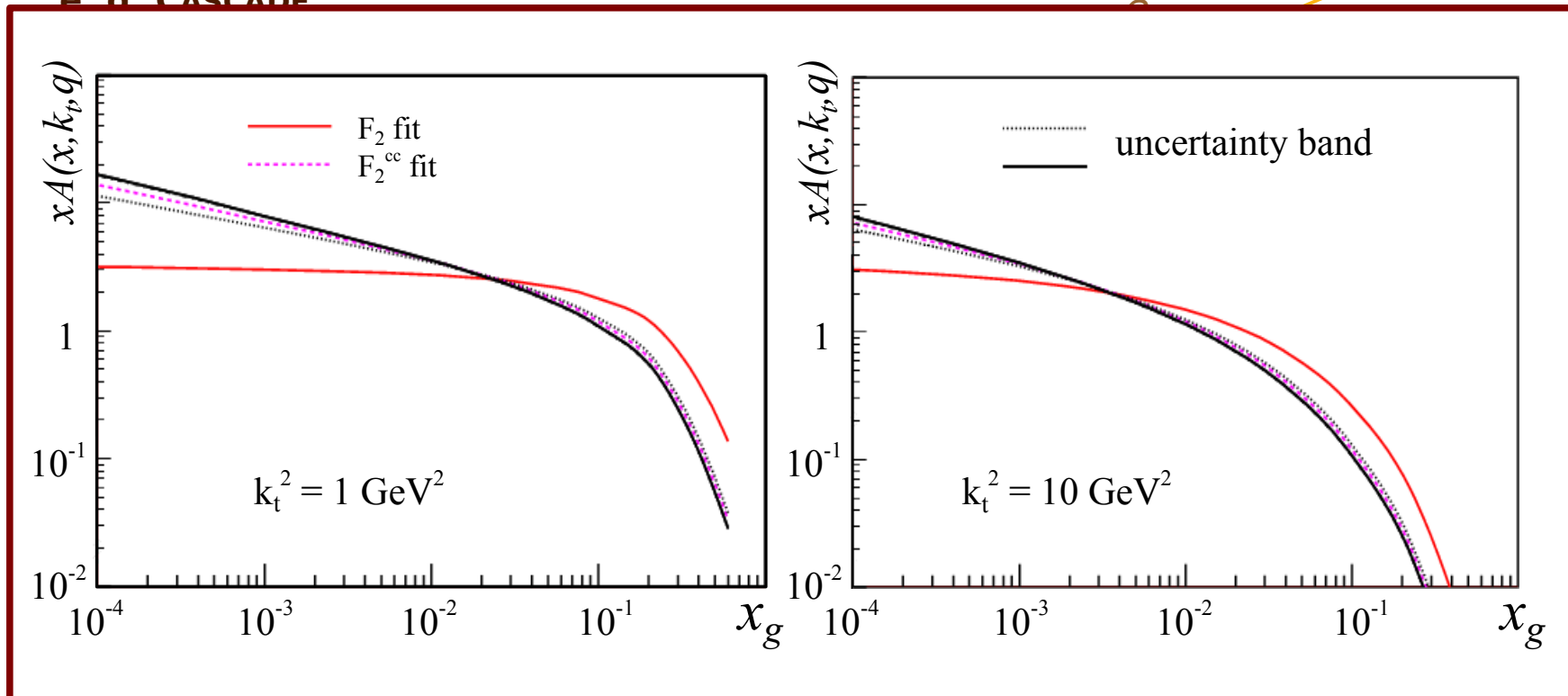
ons





Extracting Parton Distributions

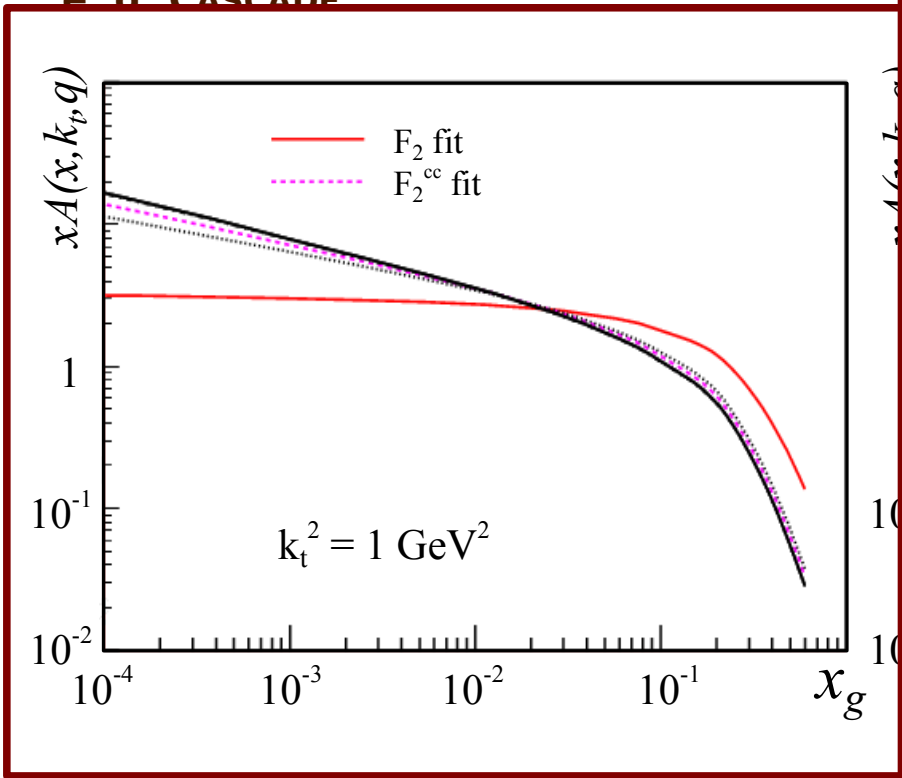
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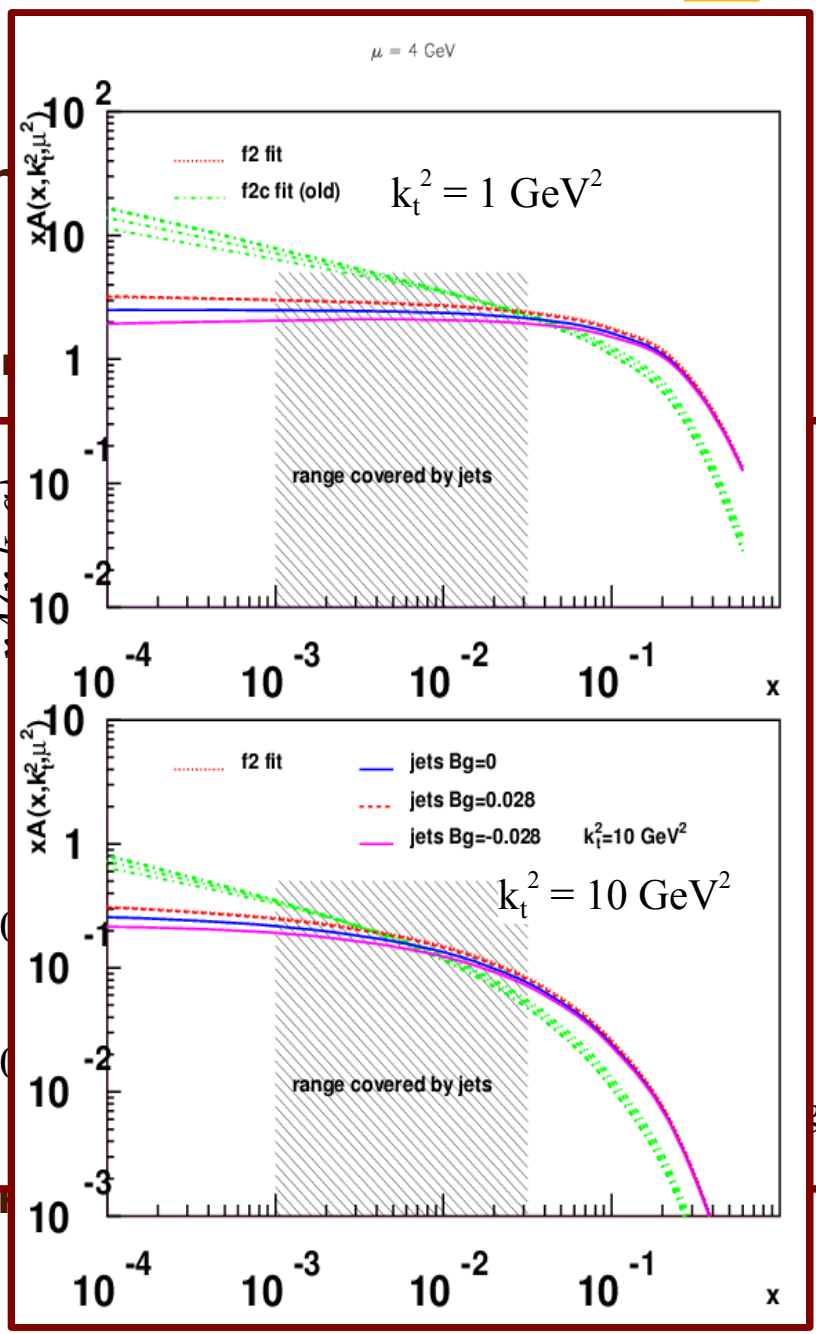
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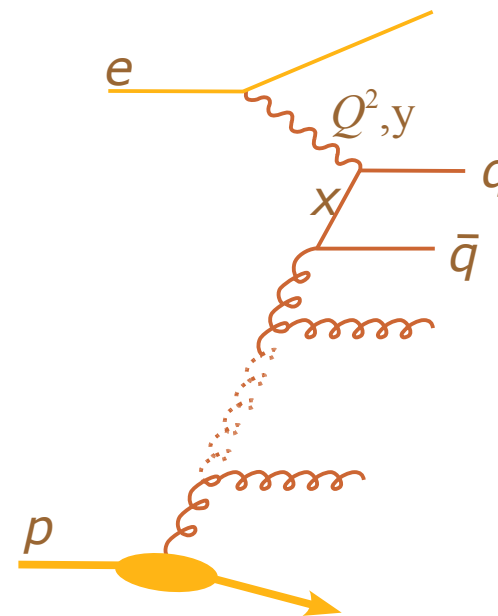




Summary

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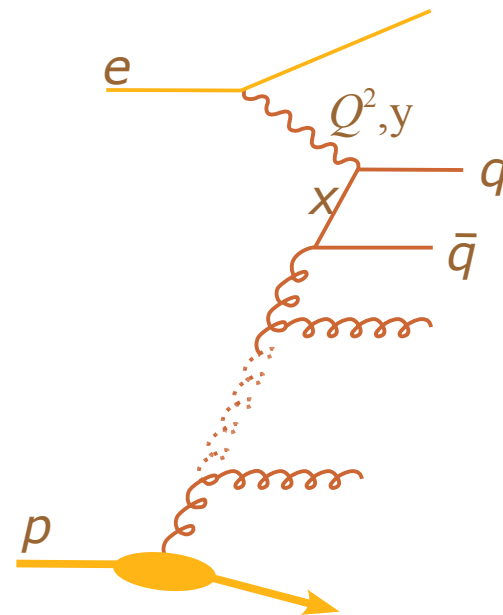
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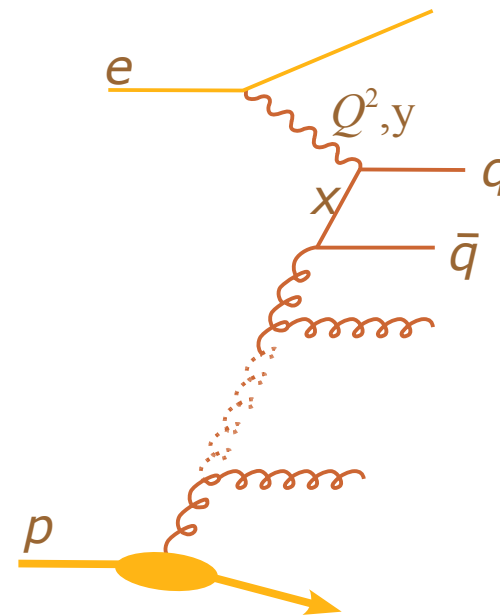
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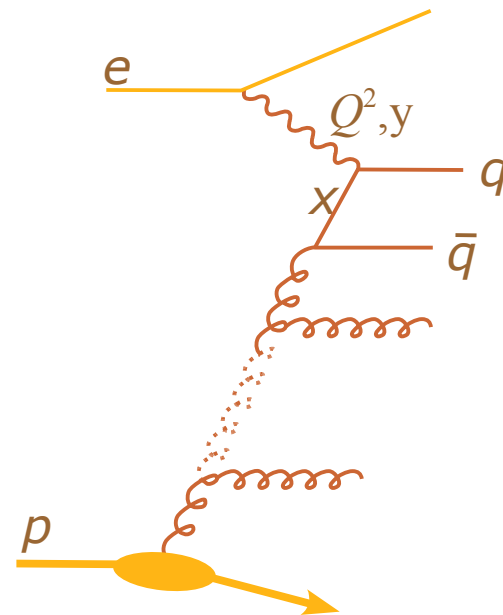
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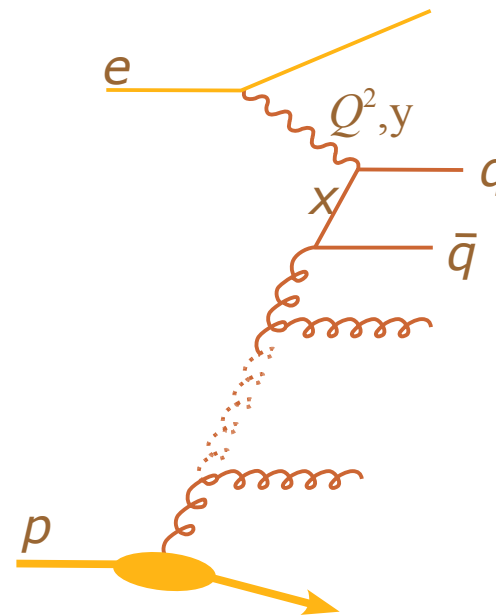
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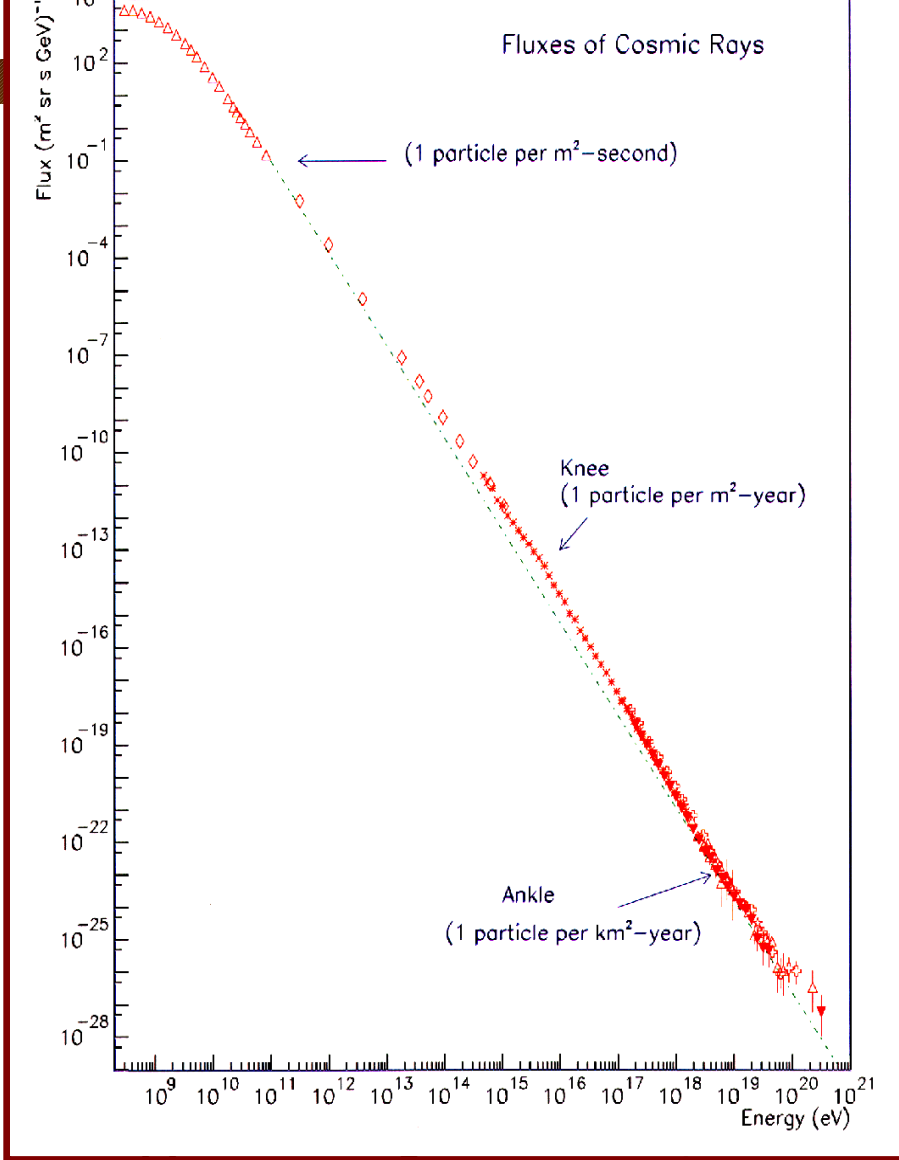
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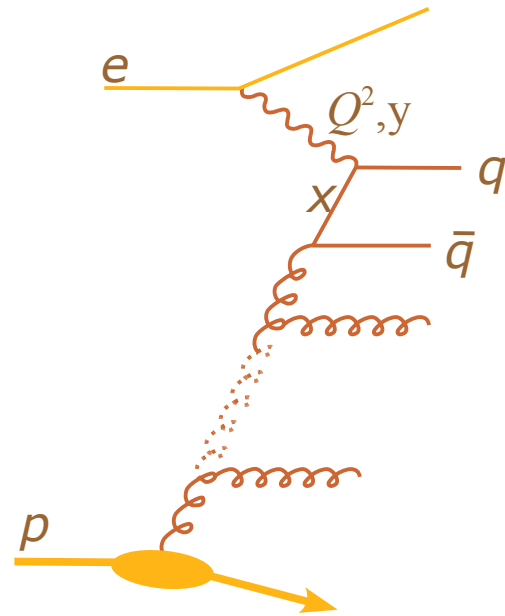
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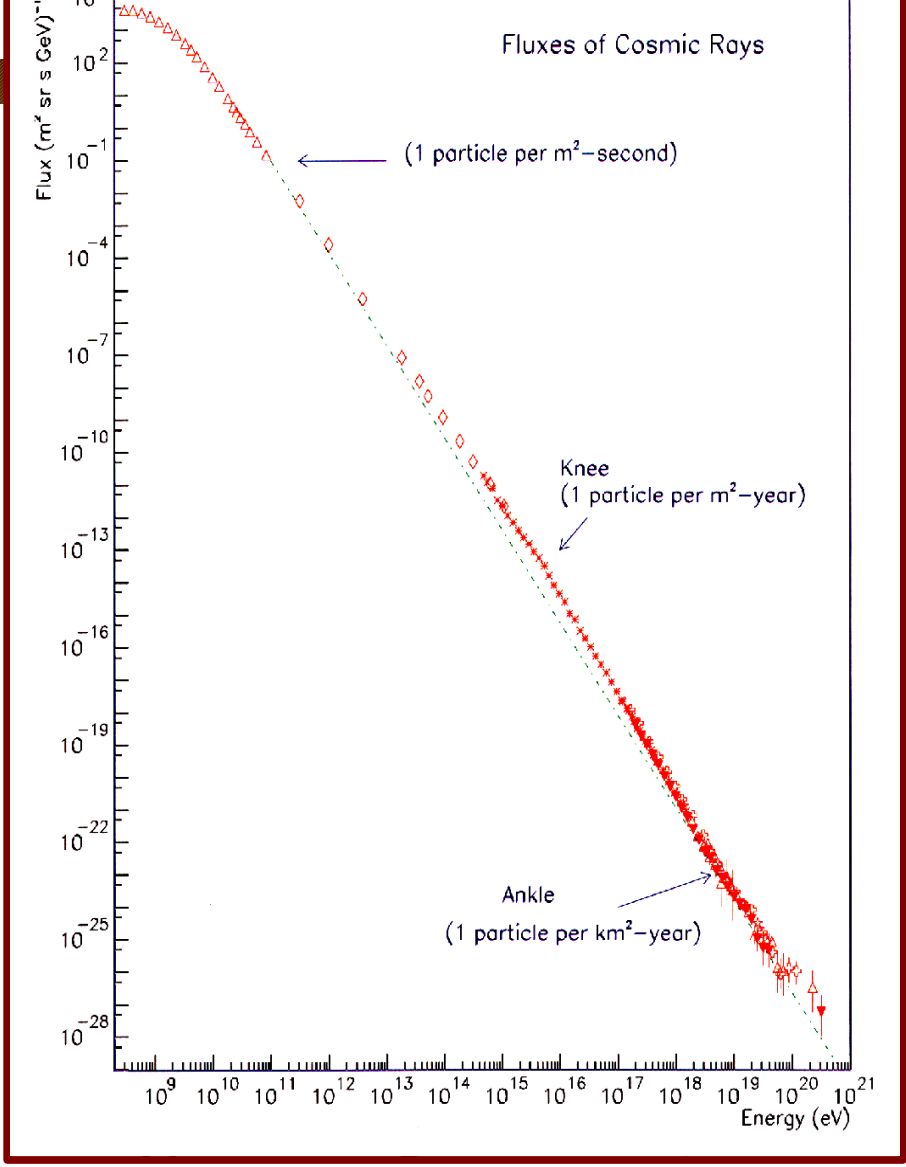
of Parton Densities in DIS



ons

χ^2 fits





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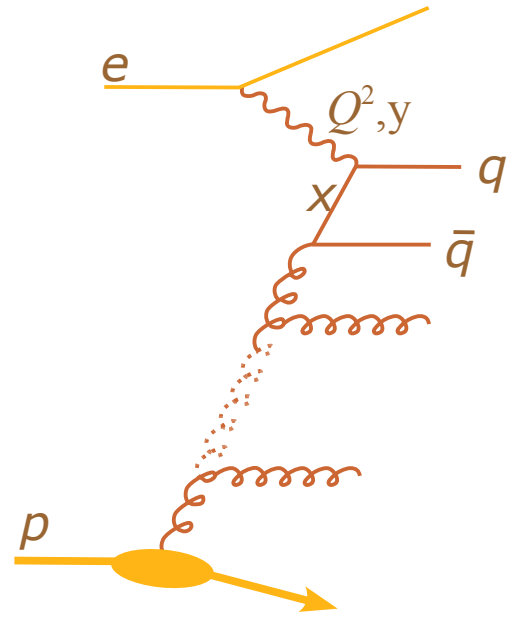
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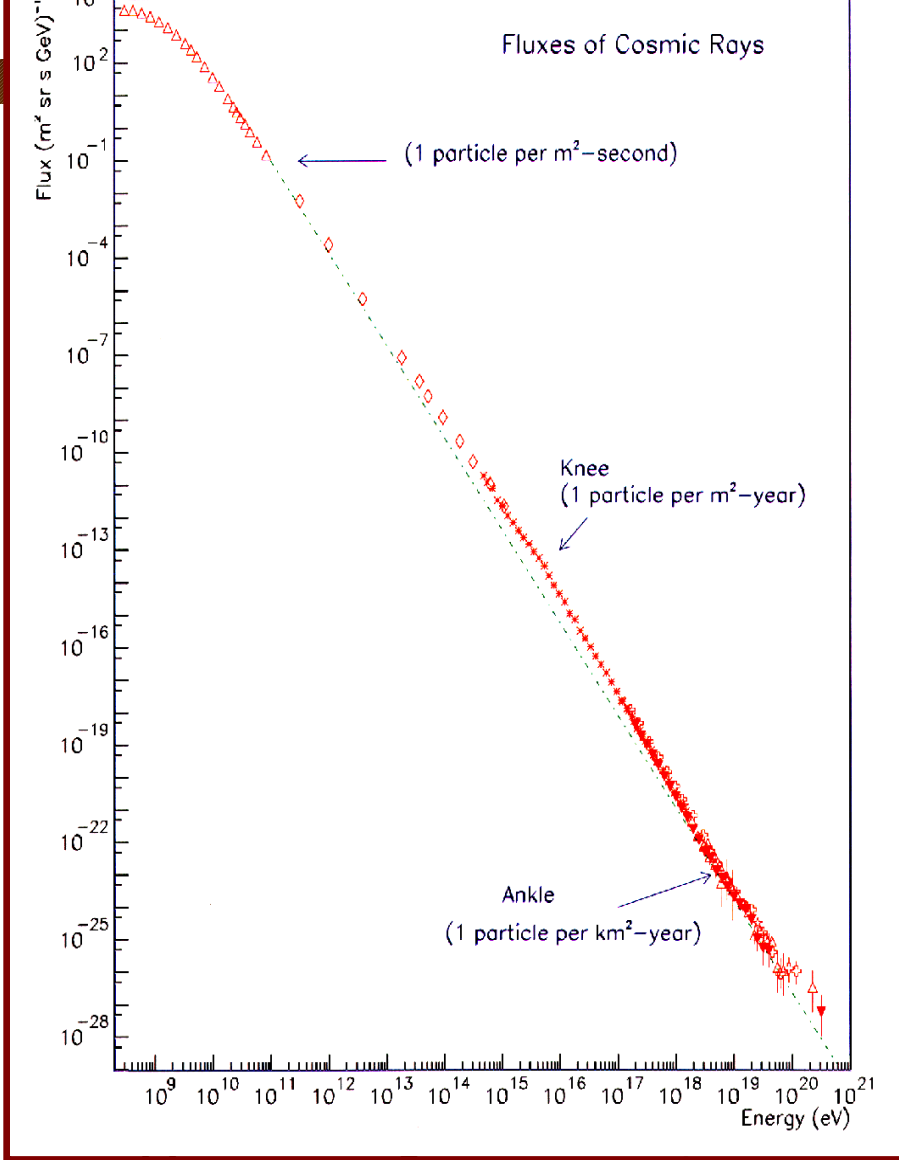
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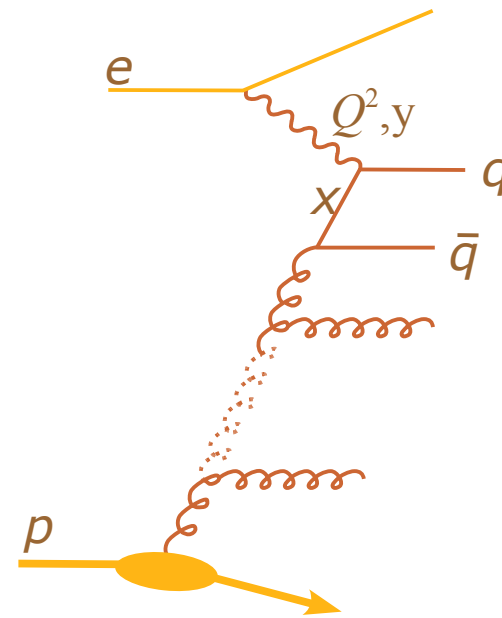


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